I Answers to Odd-Numbered Exercises

CHAPTER 1

EXERCISES 1.1 ■ PAGE 19

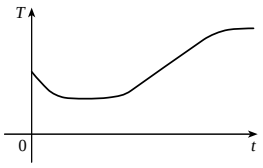
1. Yes

3. (a) 3 (b) -0.2 (c) 0, 3 (d) -0.8
(e) $[-2, 4], [-1, 3]$ (f) $[-2, 1]$

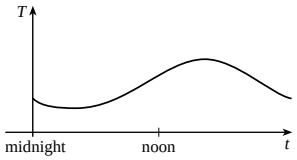
5. $[-85, 115]$ **7.** No

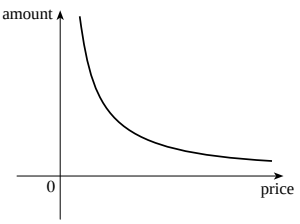
9. Yes, $[-3, 2], [-3, -2) \cup [-1, 3]$

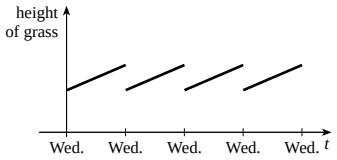
11. Diet, exercise, or illness

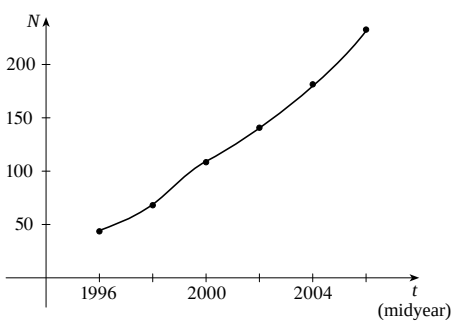
13.


15. (a) 500 MW; 730 MW (b) 4 AM; noon

17.


19.


21.


23. (a)


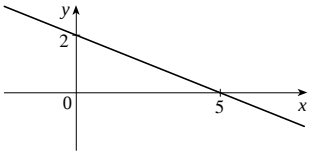
(b) 126 million; 207 million

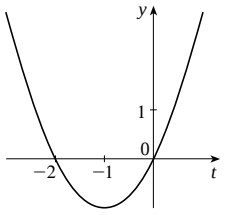
25. $12, 16, 3a^2 - a + 2, 3a^2 + a + 2, 3a^2 + 5a + 4,$
$6a^2 - 2a + 4, 12a^2 - 2a + 2, 3a^4 - a^2 + 2,$
$9a^4 - 6a^3 + 13a^2 - 4a + 4, 3a^2 + 6ah + 3h^2 - a - h + 2$

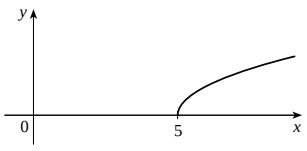
27. $-3 - h$ **29.** $-1/(ax)$

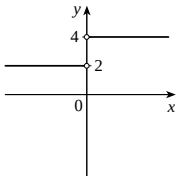
31. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ **33.** $(-\infty, \infty)$

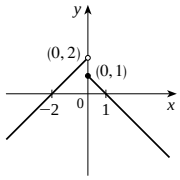
35. $(-\infty, 0) \cup (5, \infty)$ **37.** $[0, 4]$

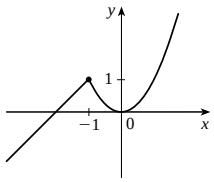
39. $(-\infty, \infty)$


41. $(-\infty, \infty)$


43. $[5, \infty)$


45. $(-\infty, 0) \cup (0, \infty)$


47. $(-\infty, \infty)$


49. $(-\infty, \infty)$


51. $f(x) = \frac{5}{2}x - \frac{11}{2}, 1 \leqslant x \leqslant 5$ **53.** $f(x) = 1 - \sqrt{-x}$

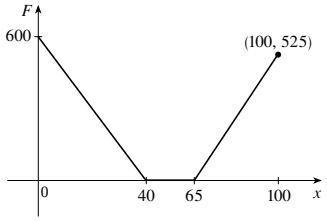
55. $f(x) = \begin{cases} -x + 3 & \text{if } 0 \leqslant x \leqslant 3 \\ 2x - 6 & \text{if } 3 < x \leqslant 5 \end{cases}$

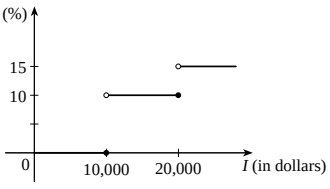
57. $A(L) = 10L - L^2, 0 < L < 10$

59. $A(x) = \sqrt{3}x^2/4, x > 0$ **61.** $S(x) = x^2 + (8/x), x > 0$

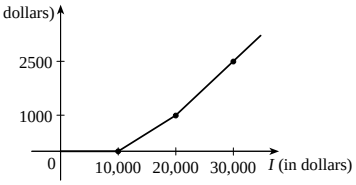
63. $V(x) = 4x^3 - 64x^2 + 240x, 0 < x < 6$

65. $F(x) = \begin{cases} 15(40 - x) & \text{if } 0 \leqslant x < 40 \\ 0 & \text{if } 40 \leqslant x \leqslant 65 \\ 15(x - 65) & \text{if } x > 65 \end{cases}$



67. (a)


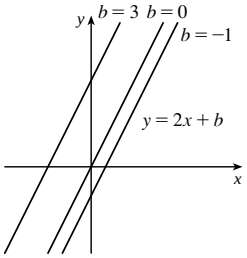
(b) \$400, \$1900

(c) 

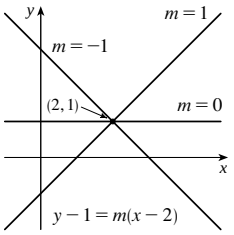
69. f is odd, g is even **71.** (a) $(-5, 3)$ (b) $(-5, -3)$
73. Odd **75.** Neither **77.** Even
79. Even; odd; neither (unless $f = 0$ or $g = 0$)

EXERCISES 1.2 ■ PAGE 33

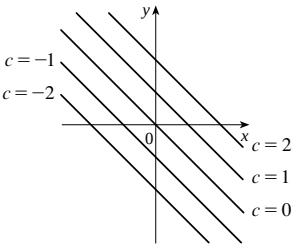
1. (a) Logarithmic (b) Root (c) Rational
(d) Polynomial, degree 2 (e) Exponential (f) Trigonometric
3. (a) h (b) f (c) g
5. (a) $y = 2x + b$, where b is the y-intercept.



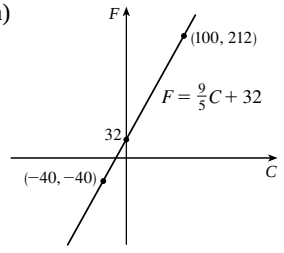
(b) $y = mx + 1 - 2m$, where m is the slope.
(c) $y = 2x - 3$



7. Their graphs have slope -1.

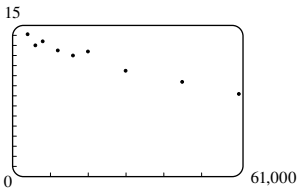


9. $f(x) = -3x(x + 1)(x - 2)$
11. (a) 8.34, change in mg for every 1 year change
(b) 8.34 mg
13. (a)



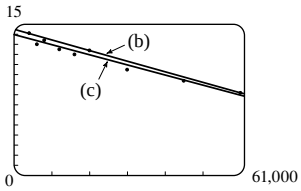
(b) $\frac{9}{5}$, change in °F for every 1°C change; 32, Fahrenheit temperature corresponding to 0°C

15. (a) $T = \frac{1}{6}N + \frac{307}{6}$ (b) $\frac{1}{6}$, change in °F for every chirp per minute change (c) 76°F
17. (a) $P = 0.434d + 15$ (b) 196 ft
19. (a) Cosine (b) Linear
21. (a)

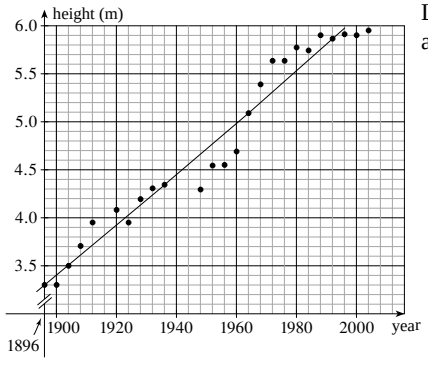


Linear model is appropriate.

(b) $y = -0.000105x + 14.521$
(c) $y = -0.00009979x + 13.951$



(d) About 11.5 per 100 population (e) About 6% (f) No
23. (a)

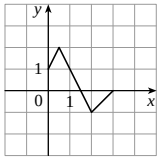


Linear model is appropriate.

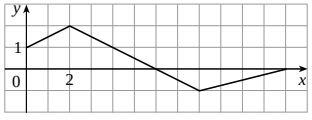
(b) $y = 0.0265x - 46.8759$ (c) 6.27 m; higher (d) No
25. Four times as bright
27. (a) $N = 3.1046A^{0.308}$ (b) 18

EXERCISES 1.3 ■ PAGE 42

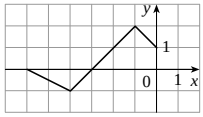
1. (a) $y = f(x) + 3$ (b) $y = f(x) - 3$ (c) $y = f(x - 3)$
(d) $y = f(x + 3)$ (e) $y = -f(x)$ (f) $y = f(-x)$
(g) $y = 3f(x)$ (h) $y = \frac{1}{3}f(x)$
3. (a) 3 (b) 1 (c) 4 (d) 5 (e) 2
5. (a)



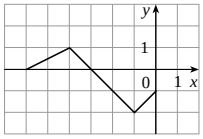
(b)



(c)

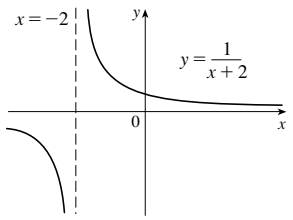


(d)

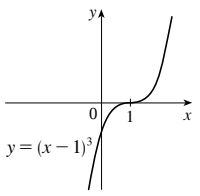


7. $y = -\sqrt{-x^2 - 5x - 4} - 1$

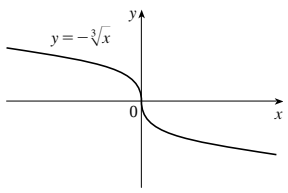
9.



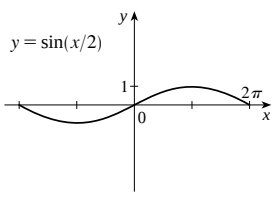
11.



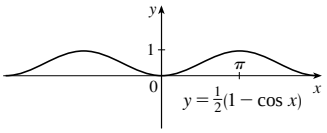
13.



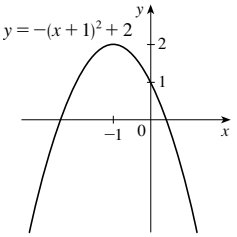
15.



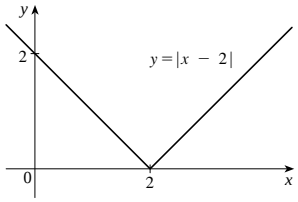
17.



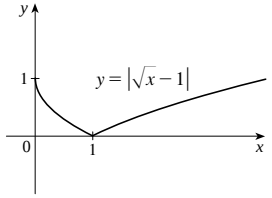
19.



21.



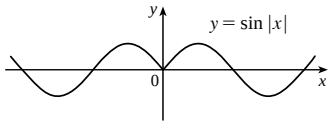
23.



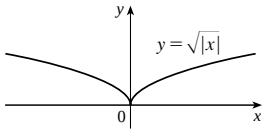
25. $L(t) = 12 + 2\sin\left[\dfrac{2\pi}{365}(t - 80)\right]$

27. (a) The portion of the graph of $y = f(x)$ to the right of the y-axis is reflected about the y-axis.

(b)



(c)



29. (a) $(f + g)(x) = x^3 + 5x^2 - 1, (-\infty, \infty)$
(b) $(f - g)(x) = x^3 - x^2 + 1, (-\infty, \infty)$
(c) $(fg)(x) = 3x^5 + 6x^4 - x^3 - 2x^2, (-\infty, \infty)$
(d) $(f/g)(x) = \dfrac{x^3 + 2x^2}{3x^2 - 1}, \{x \mid x \neq \pm 1/\sqrt{3}\}$

31. (a) $(f \circ g)(x) = 4x^2 + 4x, (-\infty, \infty)$
(b) $(g \circ f)(x) = 2x^2 - 1, (-\infty, \infty)$
(c) $(f \circ f)(x) = x^4 - 2x^2, (-\infty, \infty)$
(d) $(g \circ g)(x) = 4x + 3, (-\infty, \infty)$

33. (a) $(f \circ g)(x) = 1 - 3\cos x, (-\infty, \infty)$
(b) $(g \circ f)(x) = \cos(1 - 3x), (-\infty, \infty)$
(c) $(f \circ f)(x) = 9x - 2, (-\infty, \infty)$
(d) $(g \circ g)(x) = \cos(\cos x), (-\infty, \infty)$

35. (a) $(f \circ g)(x) = \dfrac{2x^2 + 6x + 5}{(x + 2)(x + 1)}, \{x \mid x \neq -2, -1\}$

(b) $(g \circ f)(x) = \dfrac{x^2 + x + 1}{(x + 1)^2}, \{x \mid x \neq -1, 0\}$

(c) $(f \circ f)(x) = \dfrac{x^4 + 3x^2 + 1}{x(x^2 + 1)}, \{x \mid x \neq 0\}$

(d) $(g \circ g)(x) = \dfrac{2x + 3}{3x + 5}, \left\{x \mid x \neq -2, -\frac{5}{3}\right\}$

37. $(f \circ g \circ h)(x) = 3\sin(x^2) - 2$

39. $(f \circ g \circ h)(x) = \sqrt{x^6 + 4x^3 + 1}$

41. $g(x) = 2x + x^2, f(x) = x^4$

43. $g(x) = \sqrt[3]{x}, f(x) = x/(1 + x)$

45. $g(t) = t^2, f(t) = \sec t \tan t$

47. $h(x) = \sqrt{x}, g(x) = x - 1, f(x) = \sqrt{x}$

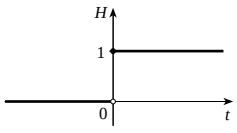
49. $h(x) = \sqrt{x}, g(x) = \sec x, f(x) = x^4$

51. (a) 4 (b) 3 (c) 0 (d) Does not exist; $f(6) = 6$ is not in the domain of g. (e) 4 (f) -2

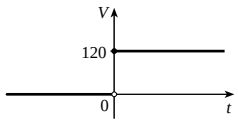
53. (a) $r(t) = 60t$ (b) $(A \circ r)(t) = 3600\pi t^2$; the area of the circle as a function of time

55. (a) $s = \sqrt{d^2 + 36}$ (b) $d = 30t$
(c) $(f \circ g)(t) = \sqrt{900t^2 + 36}$; the distance between the lighthouse and the ship as a function of the time elapsed since noon

57. (a)

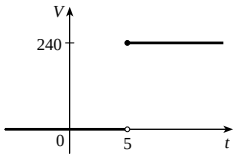


(b)



$V(t) = 120H(t)$

(c)



$V(t) = 240H(t - 5)$

59. Yes; $m_1 m_2$

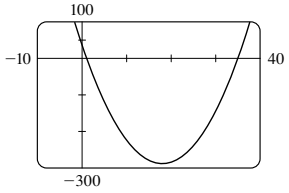
61. (a) $f(x) = x^2 + 6$ (b) $g(x) = x^2 + x - 1$

63. Yes

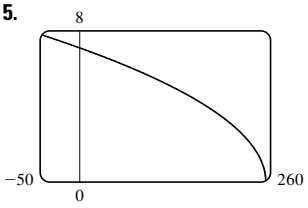
EXERCISES 1.4 ■ PAGE 50

1. (c)

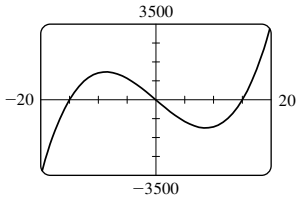
3.



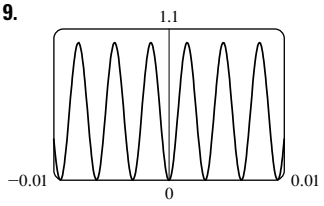
5.



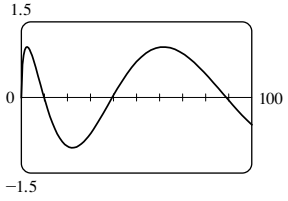
7.



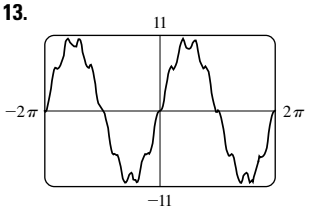
9.

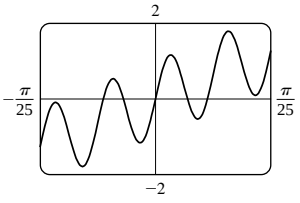


11.



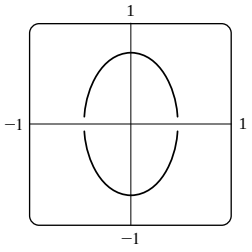
13.





15. (b) Yes; two are needed

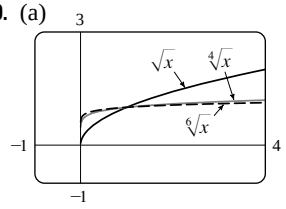
17.



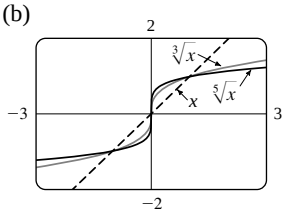
19. No **21.** −0.72, 1.22 **23.** 0.65 **25.** g

27. $-0.31 < x < 0.31$

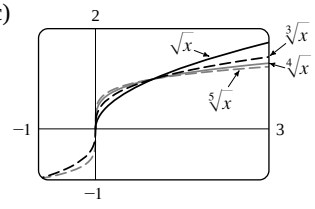
29. (a)



(b)

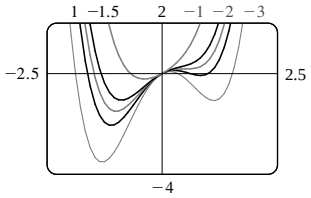


(c)



(d) Graphs of even roots are similar to $\sqrt{x}$, graphs of odd roots are similar to $\sqrt[3]{x}$. As n increases, the graph of $y = \sqrt[n]{x}$ becomes steeper near 0 and flatter for $x > 1$.

31.



If $c < -1.5$, the graph has three humps: two minimum points and a maximum point. These humps get flatter as c increases until at $c = -1.5$ two of the humps disappear and there is only one minimum point. This single hump then moves to the right and approaches the origin as c increases.

33. The hump gets larger and moves to the right.

35. If $c < 0$, the loop is to the right of the origin; if $c > 0$, the loop is to the left. The closer c is to 0, the larger the loop.

EXERCISES 1.5 ■ PAGE 57

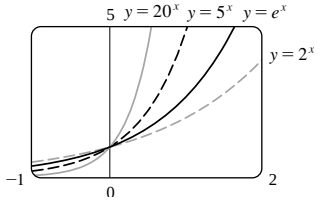
1. (a) 4 (b) $x^{-4/3}$

3. (a) $16b^{12}$ (b) $648y^7$

5. (a) $f(x) = a^x, a > 0$ (b) $\mathbb{R}$ (c) $(0, \infty)$

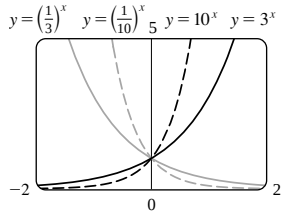
(d) See Figures 4(c), 4(b), and 4(a), respectively.

7.



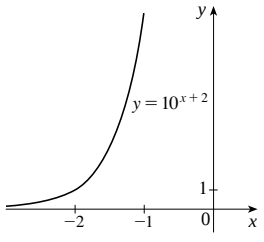
All approach 0 as $x \to -\infty$, all pass through (0, 1), and all are increasing. The larger the base, the faster the rate of increase.

9.

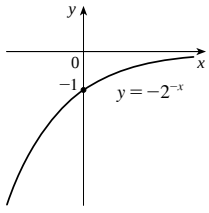


The functions with base greater than 1 are increasing and those with base less than 1 are decreasing. The latter are reflections of the former about the y-axis.

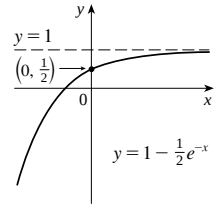
11.



13.

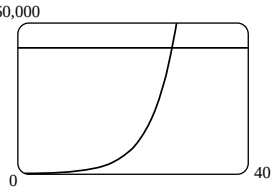


15.



17. (a) $y = e^x - 2$ (b) $y = e^{x-2}$ (c) $y = -e^x$

(d) $y = e^{-x}$ (e) $y = -e^{-x}$

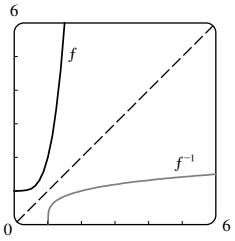
19. (a) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ (b) $(-\infty, \infty)$
21. $f(x) = 3 \cdot 2^x$ **27.** At $x \approx 35.8$
29. (a) 3200 (b) $100 \cdot 2^{t/3}$ (c) 10,159
(d) 

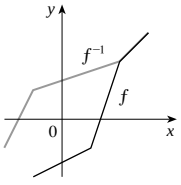
$t \approx 26.9$ h

31. $P = 2614.086(1.01693)^t$; 5381 million; 8466 million

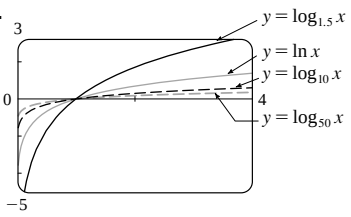
EXERCISES 1.6 ■ PAGE 69

1. (a) See Definition 1.
(b) It must pass the Horizontal Line Test.
3. No **5.** No **7.** Yes **9.** No **11.** Yes **13.** No
15. (a) 6 (b) 3 **17.** 0
19. $F = \frac{9}{5}C + 32$; the Fahrenheit temperature as a function of the Celsius temperature; $[-273.15, \infty)$
21. $y = \frac{1}{3}(x - 1)^2 - \frac{2}{3}$, $x \geqslant 1$
23. $y = \frac{1}{2}(1 + \ln x)$ **25.** $y = e^x - 3$
27. $f^{-1}(x) = \sqrt[4]{x - 1}$ **29.**



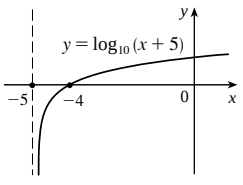


31. (a) $f^{-1}(x) = \sqrt{1 - x^2}$, $0 \leqslant x \leqslant 1$; f^{-1} and f are the same function. (b) Quarter-circle in the first quadrant
33. (a) It's defined as the inverse of the exponential function with base a, that is, $\log_a x = y \iff a^y = x$.
(b) $(0, \infty)$ (c) $\mathbb{R}$ (d) See Figure 11.
35. (a) 3 (b) -3 **37.** (a) 3 (b) -2 **39.** ln 1215
41. $\ln \dfrac{\sqrt{x}}{x + 1}$
43.

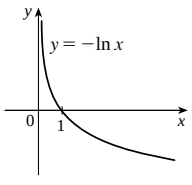


All graphs approach $-\infty$ as $x \to 0^+$, all pass through (1, 0), and all are increasing. The larger the base, the slower the rate of increase.

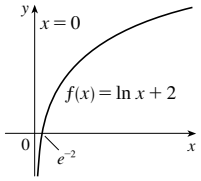
45. About 1,084,588 mi
47. (a)



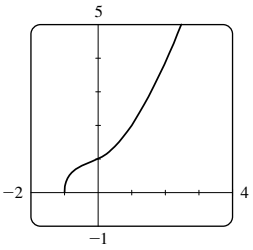
(b)



49. (a) $(0, \infty)$; $(-\infty, \infty)$ (b) e^{-2}
(c)

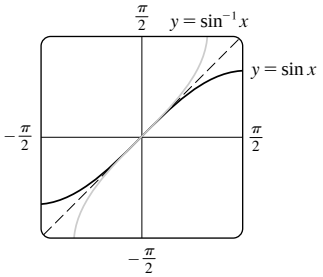


51. (a) $\frac{1}{4}(7 - \ln 6)$ (b) $\frac{1}{3}(e^2 + 10)$
53. (a) $5 + \log_2 3$ or $5 + (\ln 3)/\ln 2$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4e}\right)$
55. (a) $0 < x < 1$ (b) $x > \ln 5$
57. (a) $(\ln 3, \infty)$ (b) $f^{-1}(x) = \ln(e^x + 3)$; $\mathbb{R}$
59.



The graph passes the Horizontal Line Test.

$f^{-1}(x) = -\frac{1}{6}\sqrt[3]{4}\left(\sqrt[3]{D - 27x^2 + 20} - \sqrt[3]{D + 27x^2 - 20} + \sqrt[3]{2}\right)$, where $D = 3\sqrt{3}\sqrt{27x^4 - 40x^2 + 16}$; two of the expressions are complex.
61. (a) $f^{-1}(n) = (3/\ln 2)\ln(n/100)$; the time elapsed when there are n bacteria (b) After about 26.9 hours
63. (a) $\pi/3$ (b) π **65.** (a) $\pi/4$ (b) $\pi/4$
67. (a) 10 (b) $\pi/3$
71. $x/\sqrt{1 + x^2}$
73.



The second graph is the reflection of the first graph about the line $y = x$.

75. (a) $[-\frac{2}{3}, 0]$ (b) $[-\pi/2, \pi/2]$
77. (a) $g^{-1}(x) = f^{-1}(x) - c$ (b) $h^{-1}(x) = (1/c)f^{-1}(x)$

CHAPTER 1 REVIEW ■ PAGE 72

True-False Quiz

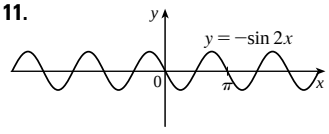
1. False **3.** False **5.** True **7.** False **9.** True
11. False **13.** False

Exercises

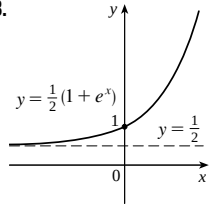
1. (a) 2.7 (b) 2.3, 5.6 (c) $[-6, 6]$ (d) $[-4, 4]$
(e) $[-4, 4]$ (f) No; it fails the Horizontal Line Test.
(g) Odd; its graph is symmetric about the origin.
3. $2a + h - 2$ **5.** $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$, $(-\infty, 0) \cup (0, \infty)$
7. $(-6, \infty)$, $\mathbb{R}$

9. (a) Shift the graph 8 units upward.
(b) Shift the graph 8 units to the left.
(c) Stretch the graph vertically by a factor of 2, then shift it 1 unit upward.
(d) Shift the graph 2 units to the right and 2 units downward.
(e) Reflect the graph about the x-axis.
(f) Reflect the graph about the line $y = x$ (assuming f is one-to-one).

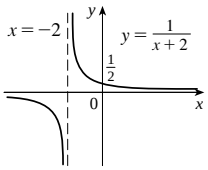
11.



13.



15.



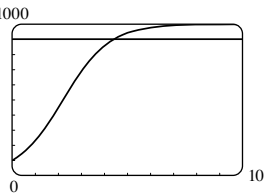
17. (a) Neither (b) Odd (c) Even (d) Neither

19. (a) $(f \circ g)(x) = \ln(x^2 - 9)$, $(-\infty, -3) \cup (3, \infty)$
(b) $(g \circ f)(x) = (\ln x)^2 - 9$, $(0, \infty)$
(c) $(f \circ f)(x) = \ln \ln x$, $(1, \infty)$
(d) $(g \circ g)(x) = (x^2 - 9)^2 - 9$, $(-\infty, \infty)$

21. $y = 0.2493x - 423.4818$; about 77.6 years

23. 1 **25.** (a) 9 (b) 2 (c) $1/\sqrt{3}$ (d) $\frac{3}{5}$

27. (a)



≈ 4.4 years

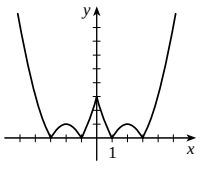
(b) $t = -\ln\left(\dfrac{1000 - P}{9P}\right)$; the time required for the population to reach a given number P.
(c) $\ln 81 \approx 4.4$ years

PRINCIPLES OF PROBLEM SOLVING ■ PAGE 80

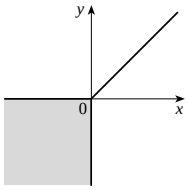
1. $a = 4\sqrt{h^2 - 16}/h$, where a is the length of the altitude and h is the length of the hypotenuse

3. $-\frac{7}{3}$, 9

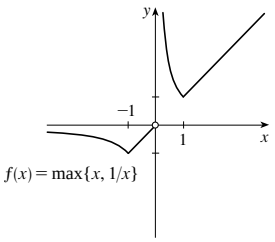
5.



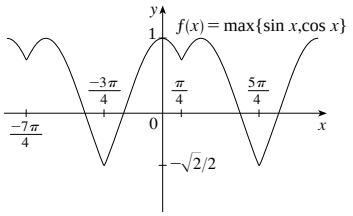
7.



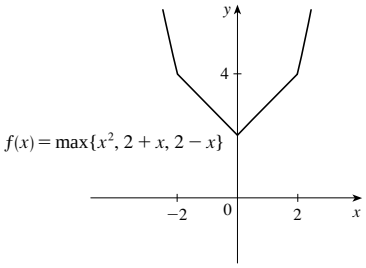
9. (a)



(b)



(c)



11. 5 **13.** $x \in [-1, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, 3]$

15. 40 mi/h **19.** $f_n(x) = x^{2^{n+1}}$

CHAPTER 2

EXERCISES 2.1 ■ PAGE 86

1. (a) -44.4, -38.8, -27.8, -22.2, $-16.\overline{6}$
(b) -33.3 (c) $-33\frac{1}{3}$

3. (a)(i) 2 (ii) 1.111111 (iii) 1.010101 (iv) 1.001001
(v) 0.666667 (vi) 0.909091 (vii) 0.990099
(viii) 0.999001 (b) 1 (c) $y = x - 3$

5. (a) (i) -32 ft/s (ii) -25.6 ft/s (iii) -24.8 ft/s
(iv) -24.16 ft/s (b) -24 ft/s

7. (a) (i) 4.65 m/s (ii) 5.6 m/s (iii) 7.55 m/s
(iv) 7 m/s (b) 6.3 m/s

9. (a) 0, 1.7321, -1.0847, -2.7433, 4.3301, -2.8173, 0, -2.1651, -2.6061, -5, 3.4202; no (c) -31.4

EXERCISES 2.2 ■ PAGE 96

1. Yes

3. (a) $\lim_{x \to -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).

(b) $\lim_{x\to 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4.

5. (a) 2 (b) 1 (c) 4 (d) Does not exist (e) 3

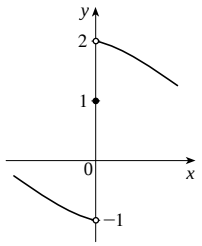
7. (a) -1 (b) -2 (c) Does not exist (d) 2 (e) 0 (f) Does not exist (g) 1 (h) 3

9. (a) $-\infty$ (b) ∞ (c) ∞ (d) $-\infty$ (e) ∞ (f) $x = -7, x = -3, x = 0, x = 6$

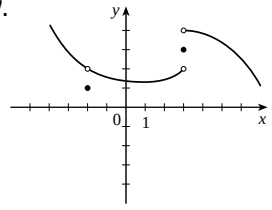
11. $\lim_{x\to a} f(x)$ exists for all a except $a = -1$.

13. (a) 1 (b) 0 (c) Does not exist

15.



17.

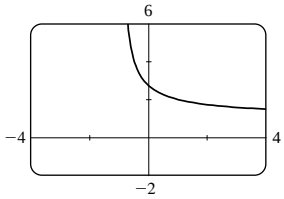


9. $\frac{2}{3}$ **21.** 5 **23.** $\frac{1}{4}$ **25.** $\frac{3}{5}$ **27.** (a) -1.5

29. $-\infty$ **31.** ∞ **33.** $-\infty$ **35.** $-\infty$ **37.** ∞

39. $-\infty$; ∞

41. (a) 2.71828 (b)



43. (a) 0.998000, 0.638259, 0.358484, 0.158680, 0.038851, 0.008928, 0.001465; 0

(b) 0.000572, -0.000614, -0.000907, -0.000978, -0.000993, -0.001000; -0.001

45. No matter how many times we zoom in toward the origin, the graph appears to consist of almost-vertical lines. This indicates more and more frequent oscillations as $x \to 0$.

47. $x \approx \pm 0.90, \pm 2.24$; $x = \pm\sin^{-1}(\pi/4), \pm(\pi - \sin^{-1}(\pi/4))$

EXERCISES 2.3 ■ PAGE 106

1. (a) -6 (b) -8 (c) 2 (d) -6 (e) Does not exist (f) 0

3. 105 **5.** $\frac{7}{8}$ **7.** 390 **9.** $\frac{3}{2}$ **11.** 4

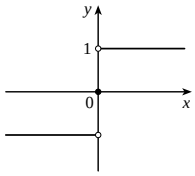
13. Does not exist **15.** $\frac{6}{5}$ **17.** -10 **19.** $\frac{1}{12}$

21. $\frac{1}{6}$ **23.** $-\frac{1}{16}$ **25.** 1 **27.** $\frac{1}{128}$ **29.** $-\frac{1}{2}$

31. $3x^2$ **33.** (a), (b) $\frac{2}{3}$ **37.** 7 **41.** 6 **43.** -4

45. Does not exist

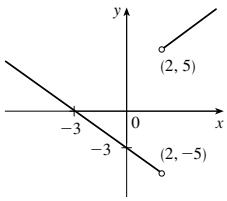
47. (a)



(b) (i) 1
(ii) -1
(iii) Does not exist
(iv) 1

49. (a) (i) 5 (ii) -5 (b) Does not exist

(c)



51. (a) (i) -2 (ii) Does not exist (iii) -3

(b) (i) $n - 1$ (ii) n (c) a is not an integer.

57. 8 **63.** 15; -1

EXERCISES 2.4 ■ PAGE 116

1. 0.1 (or any smaller positive number)

3. 1.44 (or any smaller positive number)

5. 0.0906 (or any smaller positive number)

7. 0.011 (or any smaller positive number)

9. (a) 0.031 (b) 0.010

11. (a) $\sqrt{1000/\pi}$ cm (b) Within approximately 0.0445 cm

(c) Radius; area; $\sqrt{1000/\pi}$; 1000; 5; ≈ 0.0445

13. (a) 0.025 (b) 0.0025

35. (a) 0.093 (b) $\delta = (B^{2/3} - 12)/(6B^{1/3}) - 1$, where $B = 216 + 108\varepsilon + 12\sqrt{336 + 324\varepsilon + 81\varepsilon^2}$

41. Within 0.1

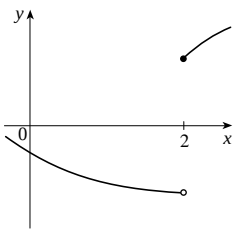
EXERCISES 2.5 ■ PAGE 127

1. $\lim_{x\to 4} f(x) = f(4)$

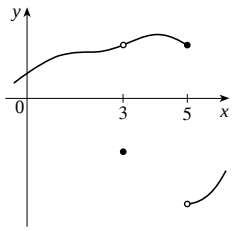
3. (a) $f(-4)$ is not defined and $\lim_{x\to a} f(x)$ [for $a = -2$, 2, and 4] does not exist

(b) -4, neither; -2, left; 2, right; 4, right

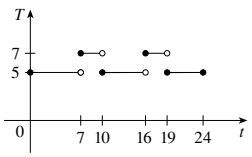
5.



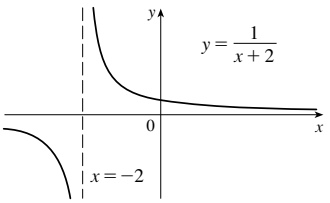
7.



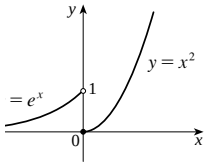
9. (a)

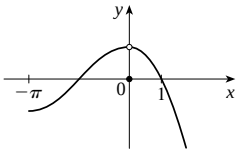


11. 4 **17.** $f(-2)$ is undefined.

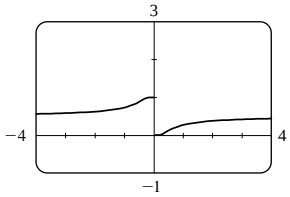


19. $\lim_{x\to 0} f(x)$ does not exist. **21.** $\lim_{x\to 0} f(x) \neq f(0)$

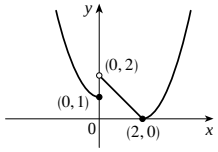


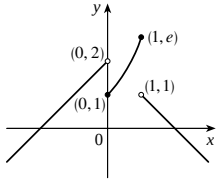


23. Define $f(2) = 3$. **25.** $(-\infty, \infty)$
27. $(-\infty, \sqrt[3]{2}) \cup (\sqrt[3]{2}, \infty)$ **29.** $[-1, 0]$
31. $(-\infty, -1] \cup (0, \infty)$
33. $x = 0$



35. $\frac{7}{3}$ **37.** 1
41. 0, left **43.** 0, right; 1, left

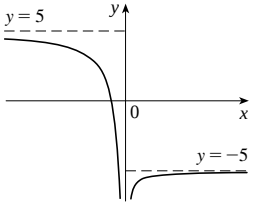




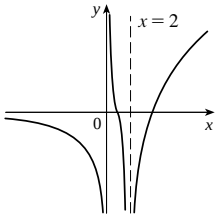
45. $\frac{2}{3}$ **47.** (a) $g(x) = x^3 + x^2 + x + 1$ (b) $g(x) = x^2 + x$
55. (b) (0.86, 0.87) **57.** (b) 70.347 **63.** None
65. Yes

EXERCISES 2.6 ■ PAGE 140

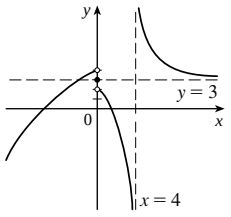
1. (a) As x becomes large, $f(x)$ approaches 5.
(b) As x becomes large negative, $f(x)$ approaches 3.
3. (a) -2 (b) 2 (c) ∞ (d) $-\infty$
(e) $x = 1$, $x = 3$, $y = -2$, $y = 2$
5.



7.

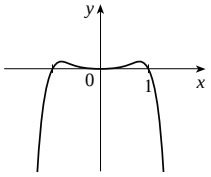


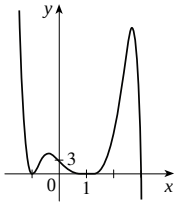
9.



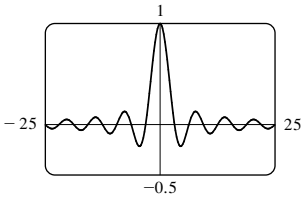
11. 0 **13.** $\frac{3}{2}$ **15.** $\frac{3}{2}$ **17.** 0 **19.** -1 **21.** 4
23. 3 **25.** $\frac{1}{6}$ **27.** $\frac{1}{2}(a - b)$ **29.** ∞
31. $-\infty$ **33.** $\pi/2$ **35.** $-\frac{1}{2}$ **37.** 0

39. (a), (b) $-\frac{1}{2}$ **41.** $y = 2$; $x = 2$
43. $y = 2$; $x = -2$, $x = 1$ **45.** $x = 5$ **47.** $y = 3$
49. $f(x) = \dfrac{2 - x}{x^2(x - 3)}$ **51.** (a) $\frac{5}{4}$ (b) 5
53. $-\infty$, $-\infty$ **55.** $-\infty$, ∞

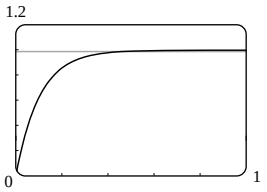




57. (a) 0 (b) An infinite number of times



59. (a) 0 (b) $\pm\infty$ **61.** 5
63. (a) v^* (b)

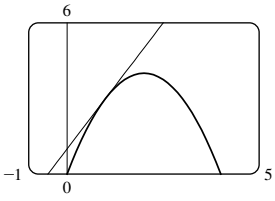


≈ 0.47 s

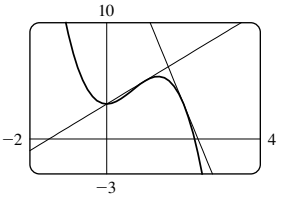
65. $N \geqslant 15$ **67.** $N \leqslant -6$, $N \leqslant -22$
69. (a) $x > 100$

EXERCISES 2.7 ■ PAGE 150

1. (a) $\dfrac{f(x) - f(3)}{x - 3}$ (b) $\lim_{x\to 3} \dfrac{f(x) - f(3)}{x - 3}$
3. (a) 2 (b) $y = 2x + 1$ (c)

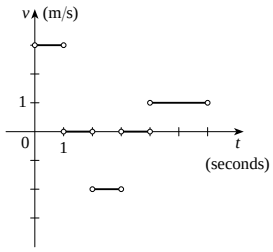


5. $y = -8x + 12$ **7.** $y = \frac{1}{2}x + \frac{1}{2}$
9. (a) $8a - 6a^2$ (b) $y = 2x + 3$, $y = -8x + 19$
(c)



11. (a) Right: $0 < t < 1$ and $4 < t < 6$; left: $2 < t < 3$; standing still: $1 < t < 2$ and $3 < t < 4$

(b)



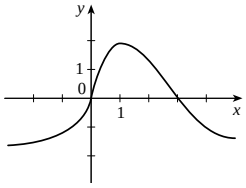
13. -24 ft/s

15. $-2/a^3$ m/s; -2 m/s; $-\frac{1}{4}$ m/s; $-\frac{2}{27}$ m/s

17. $g'(0)$, 0, $g'(4)$, $g'(2)$, $g'(-2)$

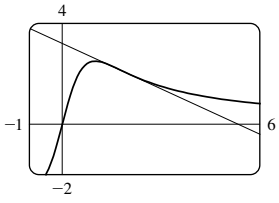
19. $f(2) = 3$; $f'(2) = 4$

21.



23. $y = 3x - 1$

25. (a) $-\frac{3}{5}$; $y = -\frac{3}{5}x + \frac{16}{5}$ (b)



27. $6a - 4$ **29.** $\dfrac{5}{(a+3)^2}$ **31.** $-\dfrac{1}{\sqrt{1-2a}}$

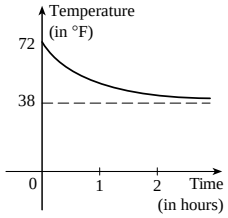
33. $f(x) = x^{10}$, $a = 1$ or $f(x) = (1+x)^{10}$, $a = 0$

35. $f(x) = 2^x$, $a = 5$

37. $f(x) = \cos x$, $a = \pi$ or $f(x) = \cos(\pi + x)$, $a = 0$

39. 1 m/s; 1 m/s

41.



Greater (in magnitude)

43. (a) (i) 23 million/year (ii) 20.5 million/year
(iii) 16 million/year
(b) 18.25 million/year (c) 17 million/year

45. (a) (i) \$20.25/unit (ii) \$20.05/unit (b) \$20/unit

47. (a) The rate at which the cost is changing per ounce of gold produced; dollars per ounce
(b) When the 800th ounce of gold is produced, the cost of production is \$17/oz.
(c) Decrease in the short term; increase in the long term

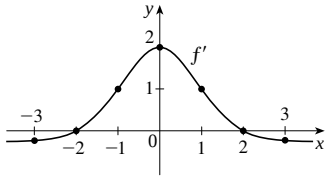
49. The rate at which the temperature is changing at 8:00 AM; 3.75°F/h

51. (a) The rate at which the oxygen solubility changes with respect to the water temperature; (mg/L)/°C
(b) $S'(16) \approx -0.25$; as the temperature increases past 16°C, the oxygen solubility is decreasing at a rate of 0.25 (mg/L)/°C.

53. Does not exist

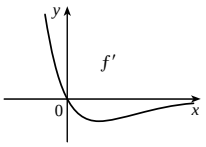
EXERCISES 2.8 ■ PAGE 162

1. (a) -0.2 (b) 0 (c) 1 (d) 2
(e) 1 (f) 0 (g) -0.2

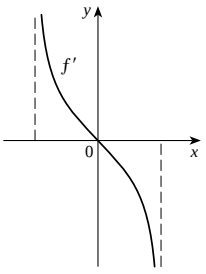


3. (a) II (b) IV (c) I (d) III

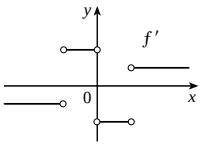
5.



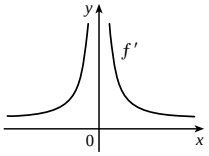
7.



9.

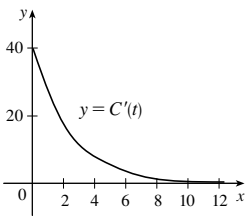


11.



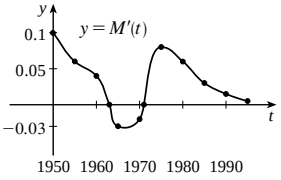
13. (a) The instantaneous rate of change of percentage of full capacity with respect to elapsed time in hours

(b)



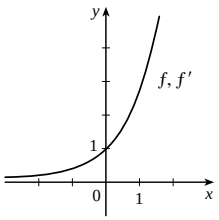
The rate of change of percentage of full capacity is decreasing and approaching 0.

15.



1963 to 1971

17. $f'(x) = e^x$



19. (a) 0, 1, 2, 4 (b) $-1, -2, -4$ (c) $f'(x) = 2x$

21. $f'(x) = \frac{1}{2}$, $\mathbb{R}$, $\mathbb{R}$ **23.** $f'(t) = 5 - 18t$, $\mathbb{R}$, $\mathbb{R}$

25. $f'(x) = 2x - 6x^2$, $\mathbb{R}$, $\mathbb{R}$

27. $g'(x) = \dfrac{-1}{2\sqrt{9-x}}$, $(-\infty, 9]$, $(-\infty, 9)$

29. $G'(t) = \dfrac{-7}{(3+t)^2}$, $(-\infty, -3) \cup (-3, \infty)$, $(-\infty, -3) \cup (-3, \infty)$

31. $f'(x) = 4x^3$, $\mathbb{R}$, $\mathbb{R}$ **33.** (a) $f'(x) = 4x^3 + 2$

35. (a) The rate at which the unemployment rate is changing, in percent unemployed per year

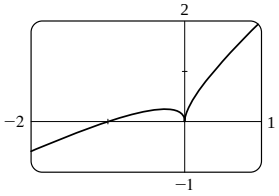
(b)

t	$U'(t)$	t	$U'(t)$
1999	−0.2	2004	−0.45
2000	0.25	2005	−0.45
2001	0.9	2006	−0.25
2002	0.65	2007	0.6
2003	−0.15	2008	1.2

37. -4 (corner); 0 (discontinuity)

39. -1 (vertical tangent); 4 (corner)

41.

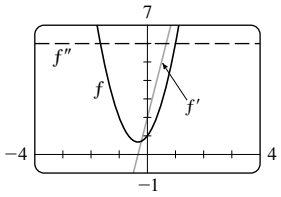


Differentiable at -1; not differentiable at 0

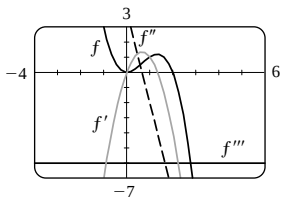
43. $a = f$, $b = f'$, $c = f''$

45. a = acceleration, b = velocity, c = position

47. $6x + 2$; 6



49.

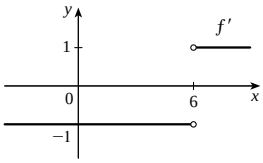


$f'(x) = 4x - 3x^2$,
$f''(x) = 4 - 6x$,
$f'''(x) = -6$,
$f^{(4)}(x) = 0$

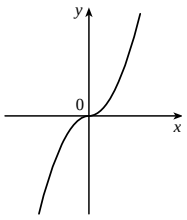
51. (a) $\frac{1}{3}a^{-2/3}$

53. $f'(x) = \begin{cases} -1 & \text{if } x < 6 \\ 1 & \text{if } x > 6 \end{cases}$

or $f'(x) = \dfrac{x-6}{|x-6|}$



55. (a)



(b) All x
(c) $f'(x) = 2|x|$

59. 63°

CHAPTER 2 REVIEW ■ PAGE 166

True-False Quiz

1. False **3.** True **5.** False **7.** True **9.** True
11. True **13.** False **15.** True **17.** True **19.** False
21. False **23.** True

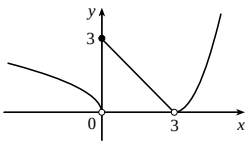
Exercises

1. (a) (i) 3 (ii) 0 (iii) Does not exist (iv) 2
(v) ∞ (vi) $-\infty$ (vii) 4 (viii) -1
(b) $y = 4$, $y = -1$ (c) $x = 0$, $x = 2$ (d) -3, 0, 2, 4

3. 1 **5.** $\frac{3}{2}$ **7.** 3 **9.** ∞ **11.** $\frac{4}{7}$ **13.** $\frac{1}{2}$

15. $-\infty$ **17.** 2 **19.** $\pi/2$ **21.** $x = 0$, $y = 0$ **23.** 1

29. (a) (i) 3 (ii) 0 (iii) Does not exist (iv) 0 (v) 0 (vi) 0
(b) At 0 and 3 (c)

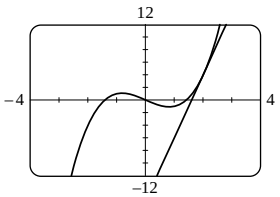


31. $\mathbb{R}$

35. (a) -8 (b) $y = -8x + 17$

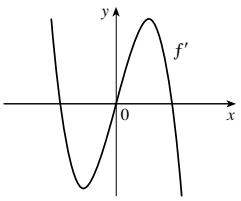
37. (a) (i) 3 m/s (ii) 2.75 m/s (iii) 2.625 m/s
(iv) 2.525 m/s (b) 2.5 m/s

39. (a) 10 (b) $y = 10x - 16$
(c)



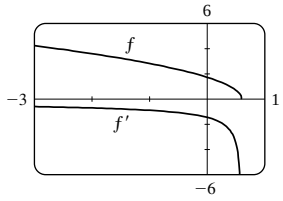
41. (a) The rate at which the cost changes with respect to the interest rate; dollars/(percent per year)
(b) As the interest rate increases past 10%, the cost is increasing at a rate of \$1200/(percent per year).
(c) Always positive

43.



45. (a) $f'(x) = -\frac{5}{2}(3 - 5x)^{-1/2}$ (b) $\left(-\infty, \frac{3}{5}\right], \left(-\infty, \frac{3}{5}\right)$

(c)



47. -4 (discontinuity), -1 (corner), 2 (discontinuity), 5 (vertical tangent)

49. The rate at which the total value of US currency in circulation is changing in billions of dollars per year; \$22.2 billion/year

51. 0

PROBLEMS PLUS ■ PAGE 170

1. $\frac{2}{3}$ **3.** -4 **5.** (a) Does not exist (b) 1

7. $a = \frac{1}{2} \pm \frac{1}{2}\sqrt{5}$ **9.** $\frac{3}{4}$ **11.** (b) Yes (c) Yes; no

13. (a) 0 (b) 1 (c) $f'(x) = x^2 + 1$

CHAPTER 3

EXERCISES 3.1 ■ PAGE 181

1. (a) See Definition of the Number e (page 180).

(b) 0.99, 1.03; $2.7 < e < 2.8$

3. $f'(x) = 0$ **5.** $f'(t) = -\frac{2}{3}$ **7.** $f'(x) = 3x^2 - 4$

9. $g'(x) = 2x - 6x^2$ **11.** $g'(t) = -\frac{3}{2}t^{-7/4}$ **13.** $A'(s) = 60/s^6$

15. $R'(a) = 18a + 6$ **17.** $S'(p) = \frac{1}{2}p^{-1/2} - 1$

19. $y' = 3e^x - \frac{4}{3}x^{-4/3}$ **21.** $h'(u) = 3Au^2 + 2Bu + C$

23. $y' = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$ **25.** $j'(x) = 2.4x^{1.4}$

27. $H'(x) = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$

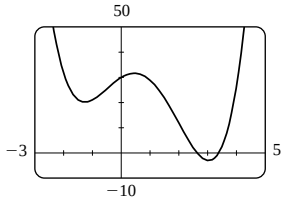
29. $u' = \frac{1}{5}t^{-4/5} + 10t^{3/2}$

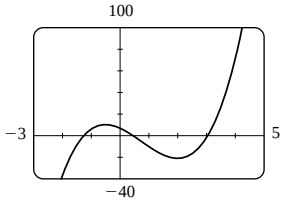
31. $z' = -10A/y^{11} + Be^y$ **33.** $y = \frac{1}{4}x + \frac{3}{4}$

35. Tangent: $y = 2x + 2$; normal: $y = -\frac{1}{2}x + 2$

37. $y = 3x - 1$ **39.** $f'(x) = 4x^3 - 6x^2 + 2x$

41. (a) (c) $4x^3 - 9x^2 - 12x + 7$





43. $f'(x) = 100x^9 + 25x^4 - 1$; $f''(x) = 900x^8 + 100x^3$

45. $f'(x) = 2 - \frac{15}{4}x^{-1/4}$, $f''(x) = \frac{15}{16}x^{-5/4}$

47. (a) $v(t) = 3t^2 - 3$, $a(t) = 6t$ (b) 12 m/s^2

(c) $a(1) = 6$ m/s^2

49. (a) $V = 5.3/P$

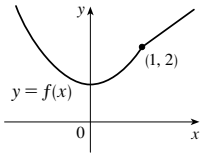
(b) -0.00212; instantaneous rate of change of the volume with respect to the pressure at 25°C; m^3/kPa

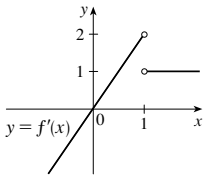
51. $(-2, 21)$, $(1, -6)$

55. $y = 12x - 15$, $y = 12x + 17$ **57.** $y = \frac{1}{3}x - \frac{1}{3}$

59. $(\pm 2, 4)$ **63.** $P(x) = x^2 - x + 3$ **65.** $y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$

67. No

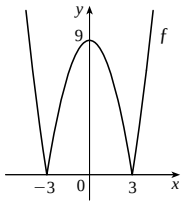


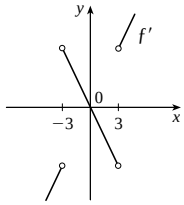


69. (a) Not differentiable at 3 or -3

$$f'(x) = \begin{cases} 2x & \text{if } |x| > 3 \\ -2x & \text{if } |x| < 3 \end{cases}$$

(b)





71. $y = 2x^2 - x$ **73.** $a = -\frac{1}{2}$, $b = 2$ **75.** $m = 4$, $b = -4$

77. 1000 **79.** 3; 1

EXERCISES 3.2 ■ PAGE 189

1. $1 - 2x + 6x^2 - 8x^3$ **3.** $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$

5. $y' = \dfrac{1 - x}{e^x}$ **7.** $g'(x) = \dfrac{10}{(3 - 4x)^2}$ **9.** $H'(u) = 2u - 1$

11. $F'(y) = 5 + \dfrac{14}{y^2} + \dfrac{9}{y^4}$

13. $y' = \dfrac{x^2(3 - x^2)}{(1 - x^2)^2}$ **15.** $y' = \dfrac{2t(-t^4 - 4t^2 + 7)}{(t^4 - 3t^2 + 1)^2}$

17. $y' = e^p\left(1 + \frac{3}{2}\sqrt{p} + p + p\sqrt{p}\right)$ **19.** $y' = 2v - 1/\sqrt{v}$

21. $f'(t) = \dfrac{4 + t^{1/2}}{\left(2 + \sqrt{t}\right)^2}$ **23.** $f'(x) = \dfrac{-ACe^x}{(B + Ce^x)^2}$

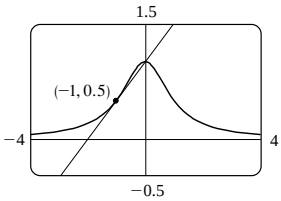
25. $f'(x) = \dfrac{2cx}{(x^2 + c)^2}$

27. $(x^4 + 4x^3)e^x$; $(x^4 + 8x^3 + 12x^2)e^x$

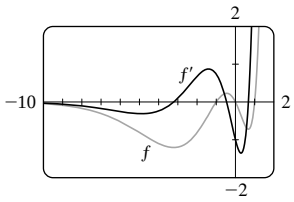
29. $\dfrac{2x^2 + 2x}{(1 + 2x)^2}$; $\dfrac{2}{(1 + 2x)^3}$ **31.** $y = \frac{2}{3}x - \frac{2}{3}$

33. $y = 2x$; $y = -\frac{1}{2}x$

35. (a) $y = \frac{1}{2}x + 1$ (b)

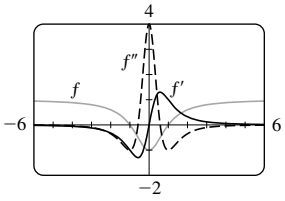


37. (a) $e^x(x^3 + 3x^2 - x - 1)$ (b)



39. (a) $f'(x) = \dfrac{4x}{(x^2+1)^2}$; $f''(x) = \dfrac{4(1-3x^2)}{(x^2+1)^3}$

(b)



41. $\frac{1}{4}$ **43.** (a) -16 (b) $-\frac{20}{9}$ (c) 20 **45.** 7

47. $y = -2x + 18$

49. (a) 0 (b) $-\frac{2}{3}$

51. (a) $y' = xg'(x) + g(x)$

(b) $y' = \dfrac{g(x) - xg'(x)}{[g(x)]^2}$ (c) $y' = \dfrac{xg'(x) - g(x)}{x^2}$

53. Two, $\left(-2 \pm \sqrt{3}, \frac{1}{2}\left(1 \mp \sqrt{3}\right)\right)$ **55.** 1

57. \$1.627 billion/year **59.** (c) $3e^{3x}$

61. $f'(x) = (x^2 + 2x)e^x$, $f''(x) = (x^2 + 4x + 2)e^x$, $f'''(x) = (x^2 + 6x + 6)e^x$, $f^{(4)}(x) = (x^2 + 8x + 12)e^x$, $f^{(5)}(x) = (x^2 + 10x + 20)e^x$; $f^{(n)}(x) = [x^2 + 2nx + n(n-1)]e^x$

EXERCISES 3.3 ■ PAGE 197

1. $f'(x) = 6x + 2\sin x$ **3.** $f'(x) = \cos x - \frac{1}{2}\csc^2 x$

5. $y' = \sec\theta\,(\sec^2\theta + \tan^2\theta)$

7. $y' = -c\sin t + t(t\cos t + 2\sin t)$

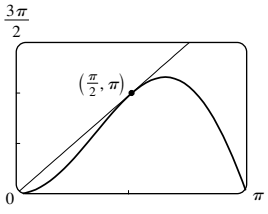
9. $y' = \dfrac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}$ **11.** $f'(\theta) = \dfrac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$

13. $y' = \dfrac{(t^2+t)\cos t + \sin t}{(1+t)^2}$

15. $f'(x) = e^x \csc x\,(-x\cot x + x + 1)$

21. $y = 2\sqrt{3}x - \frac{2}{3}\sqrt{3}\pi + 2$ **23.** $y = x - \pi - 1$

25. (a) $y = 2x$ (b)



27. (a) $\sec x\tan x - 1$

29. $\theta\cos\theta + \sin\theta$; $2\cos\theta - \theta\sin\theta$

31. (a) $f'(x) = (1 + \tan x)/\sec x$ (b) $f'(x) = \cos x + \sin x$

33. $(2n+1)\pi \pm \frac{1}{3}\pi$, n an integer

35. (a) $v(t) = 8\cos t$, $a(t) = -8\sin t$

(b) $4\sqrt{3}$, -4, $-4\sqrt{3}$; to the left

37. 5 ft/rad **39.** 3 **41.** 3 **43.** $-\frac{3}{4}$

45. $\frac{1}{2}$ **47.** $-\sqrt{2}$ **49.** $-\cos x$ **51.** $A = -\frac{3}{10}$, $B = -\frac{1}{10}$

53. (a) $\sec^2 x = \dfrac{1}{\cos^2 x}$ (b) $\sec x\tan x = \dfrac{\sin x}{\cos^2 x}$

(c) $\cos x - \sin x = \dfrac{\cot x - 1}{\csc x}$

55. 1

EXERCISES 3.4 ■ PAGE 205

1. $\dfrac{4}{3\sqrt[3]{(1+4x)^2}}$ **3.** $\pi\sec^2\pi x$ **5.** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

7. $F'(x) = 10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)$

9. $F'(x) = -\dfrac{1}{\sqrt{1-2x}}$ **11.** $f'(z) = -\dfrac{2z}{(z^2+1)^2}$

13. $y' = -3x^2\sin(a^3 + x^3)$ **15.** $y' = e^{-kx}(-kx + 1)$

17. $f'(x) = (2x-3)^3(x^2+x+1)^4(28x^2 - 12x - 7)$

19. $h'(t) = \frac{2}{3}(t+1)^{-1/3}(2t^2-1)^2(20t^2 + 18t - 1)$

21. $y' = \dfrac{-12x(x^2+1)^2}{(x^2-1)^4}$ **23.** $y' = \dfrac{3e^{3x}}{\sqrt{1+2e^{3x}}}$

25. $y' = 5^{-1/x}(\ln 5)/x^2$ **27.** $y' = (r^2+1)^{-3/2}$

29. $F'(t) = e^{t\sin 2t}(2t\cos 2t + \sin 2t)$

31. $y' = 2\cos(\tan 2x)\sec^2(2x)$ **33.** $y' = 2^{\sin\pi x}(\pi\ln 2)\cos\pi x$

35. $y' = \dfrac{4e^{2x}}{(1+e^{2x})^2}\sin\dfrac{1-e^{2x}}{1+e^{2x}}$

37. $y' = -2\cos\theta\cot(\sin\theta)\csc^2(\sin\theta)$

39. $f'(t) = \sec^2(e^t)e^t + e^{\tan t}\sec^2 t$

41. $f'(t) = 4\sin(e^{\sin^2 t})\cos(e^{\sin^2 t})\,e^{\sin^2 t}\sin t\cos t$

43. $g'(x) = 2r^2 p(\ln a)(2ra^{rx} + n)^{p-1}a^{rx}$

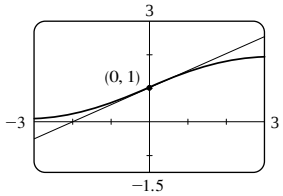
45. $y' = \dfrac{-\pi\cos(\tan\pi x)\sec^2(\pi x)\sin\sqrt{\sin(\tan\pi x)}}{2\sqrt{\sin(\tan\pi x)}}$

47. $y' = -2x\sin(x^2)$; $y'' = -4x^2\cos(x^2) - 2\sin(x^2)$

49. $e^{\alpha x}(\beta\cos\beta x + \alpha\sin\beta x)$; $e^{\alpha x}[(\alpha^2 - \beta^2)\sin\beta x + 2\alpha\beta\cos\beta x]$

51. $y = 20x + 1$ **53.** $y = -x + \pi$

55. (a) $y = \frac{1}{2}x + 1$ (b)



57. (a) $f'(x) = \dfrac{2 - 2x^2}{\sqrt{2 - x^2}}$

59. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n an integer

61. 24 **63.** (a) 30 (b) 36

65. (a) $\frac{3}{4}$ (b) Does not exist (c) -2

67. $-\frac{1}{6}\sqrt{2}$

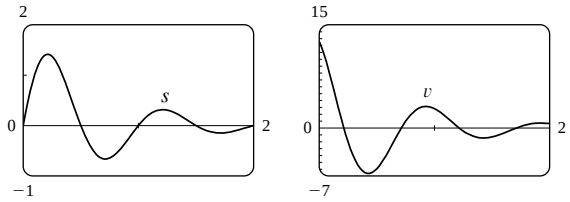
69. (a) $F'(x) = e^x f'(e^x)$ (b) $G'(x) = e^{f(x)}f'(x)$

71. 120 **73.** 96

77. $-2^{50}\cos 2x$ **79.** $v(t) = \frac{5}{2}\pi\cos(10\pi t)$ cm/s

81. (a) $\dfrac{dB}{dt} = \dfrac{7\pi}{54}\cos\dfrac{2\pi t}{5.4}$ (b) 0.16

83. $v(t) = 2e^{-1.5t}(2\pi\cos 2\pi t - 1.5\sin 2\pi t)$



85. dv/dt is the rate of change of velocity with respect to time; dv/ds is the rate of change of velocity with respect to displacement

87. (a) $y = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$
(b) $-670.63\ \mu A$

89. (b) The factored form **93.** (b) $-n\cos^{n-1}x\sin[(n+1)x]$

EXERCISES 3.5 ■ PAGE 215

1. (a) $y' = 9x/y$ (b) $y = \pm\sqrt{9x^2-1}$, $y' = \pm 9x/\sqrt{9x^2-1}$

3. (a) $y' = -y^2/x^2$ (b) $y = x/(x-1)$, $y' = -1/(x-1)^2$

5. $y' = -\dfrac{x^2}{y^2}$ **7.** $y' = \dfrac{2x+y}{2y-x}$

9. $y' = \dfrac{3y^2 - 5x^4 - 4x^3y}{x^4 + 3y^2 - 6xy}$ **11.** $y' = \dfrac{2x + y\sin x}{\cos x - 2y}$

13. $y' = \tan x \tan y$ **15.** $y' = \dfrac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$

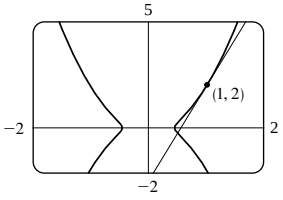
17. $y' = \dfrac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$

19. $y' = \dfrac{e^y\sin x + y\cos(xy)}{e^y\cos x - x\cos(xy)}$ **21.** $-\frac{16}{13}$

23. $x' = \dfrac{-2x^4y + x^3 - 6xy^2}{4x^3y^2 - 3x^2y + 2y^3}$ **25.** $y = \frac{1}{2}x$

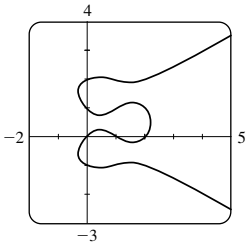
27. $y = -x + 2$ **29.** $y = x + \frac{1}{2}$ **31.** $y = -\frac{9}{13}x + \frac{40}{13}$

33. (a) $y = \frac{9}{2}x - \frac{5}{2}$ (b)



35. $-81/y^3$ **37.** $-2x/y^5$ **39.** $1/e^2$

41. (a)



Eight; $x \approx 0.42, 1.58$

(b) $y = -x + 1$, $y = \frac{1}{3}x + 2$ (c) $1 \mp \frac{1}{3}\sqrt{3}$

43. $\left(\pm\frac{5}{4}\sqrt{3}, \pm\frac{5}{4}\right)$ **45.** $(x_0x/a^2) - (y_0y/b^2) = 1$

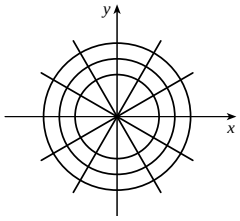
49. $y' = \dfrac{2\tan^{-1}x}{1+x^2}$ **51.** $y' = \dfrac{1}{\sqrt{-x^2 - x}}$

53. $G'(x) = -1 - \dfrac{x\arccos x}{\sqrt{1-x^2}}$ **55.** $h'(t) = 0$

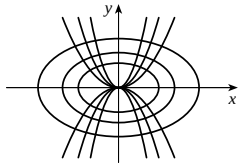
57. $y' = \sin^{-1}x$ **59.** $y' = \dfrac{\sqrt{a^2 - b^2}}{a + b\cos x}$

61. $1 - \dfrac{x\arcsin x}{\sqrt{1-x^2}}$

65.



67.



71. (a) $\dfrac{V^3(nb - V)}{PV^3 - n^2aV + 2n^3ab}$ (b) -4.04 L/atm

73. $\left(\pm\sqrt{3}, 0\right)$ **75.** $(-1,-1), (1,1)$ **77.** (b) $\frac{3}{2}$

79. (a) 0 (b) $-\frac{1}{2}$

EXERCISES 3.6 ■ PAGE 223

1. The differentiation formula is simplest.

3. $f'(x) = \dfrac{\cos(\ln x)}{x}$ **5.** $f'(x) = -\dfrac{1}{x}$

7. $f'(x) = \dfrac{3x^2}{(x^3+1)\ln 10}$ **9.** $f'(x) = \dfrac{\sin x}{x} + \cos x\ln(5x)$

11. $g'(x) = \dfrac{2x^2 - 1}{x(x^2-1)}$ **13.** $G'(y) = \dfrac{10}{2y+1} - \dfrac{y}{y^2+1}$

15. $F'(s) = \dfrac{1}{s\ln s}$ **17.** $y' = \sec^2(\ln(ax+b))\dfrac{a}{ax+b}$

19. $y' = \dfrac{-x}{1+x}$ **21.** $y' = \dfrac{1}{\ln 10} + \log_{10}x$

23. $y' = x + 2x\ln(2x)$; $y'' = 3 + 2\ln(2x)$

25. $y' = \dfrac{1}{\sqrt{1+x^2}}$; $y'' = \dfrac{-x}{(1+x^2)^{3/2}}$

27. $f'(x) = \dfrac{2x - 1 - (x-1)\ln(x-1)}{(x-1)[1 - \ln(x-1)]^2}$;
$(1, 1+e) \cup (1+e, \infty)$

29. $f'(x) = \dfrac{2(x-1)}{x(x-2)}$; $(-\infty, 0) \cup (2, \infty)$ **31.** 1

33. $y = 3x - 9$ **35.** $\cos x + 1/x$ **37.** 7

39. $y' = (x^2+2)^2(x^4+4)^4\left(\dfrac{4x}{x^2+2} + \dfrac{16x^3}{x^4+4}\right)$

41. $y' = \sqrt{\dfrac{x-1}{x^4+1}}\left(\dfrac{1}{2x-2} - \dfrac{2x^3}{x^4+1}\right)$

43. $y' = x^x(1 + \ln x)$

45. $y' = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x\ln x\right)$

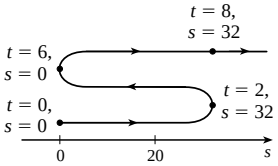
47. $y' = (\cos x)^x(-x\tan x + \ln\cos x)$

49. $y' = (\tan x)^{1/x}\left(\dfrac{\sec^2 x}{x\tan x} - \dfrac{\ln\tan x}{x^2}\right)$

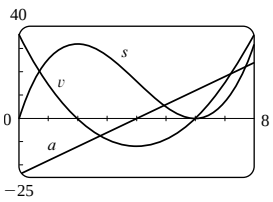
51. $y' = \dfrac{2x}{x^2 + y^2 - 2y}$ **53.** $f^{(n)}(x) = \dfrac{(-1)^{n-1}(n-1)!}{(x-1)^n}$

EXERCISES 3.7 ■ PAGE 233

1. (a) $3t^2 - 24t + 36$ (b) -9 ft/s (c) $t = 2, 6$
(d) $0 \leqslant t < 2$, $t > 6$ (e) 96 ft
(f)

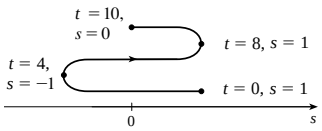


(g) $6t - 24$; -6 ft/s^2
(h)

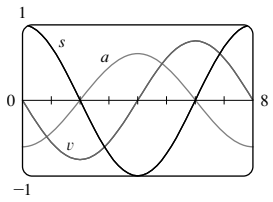


(i) Speeding up when $2 < t < 4$ or $t > 6$; slowing down when $0 \leqslant t < 2$ or $4 < t < 6$

3. (a) $-\frac{\pi}{4}\sin\left(\frac{\pi t}{4}\right)$ (b) $-\frac{1}{8}\pi\sqrt{2}$ ft/s (c) $t = 0, 4, 8$
(d) $4 < t < 8$ (e) 4 ft
(f)



(g) $-\frac{1}{16}\pi^2\cos(\pi t/4)$; $\frac{1}{32}\pi^2\sqrt{2}$ ft/s^2
(h)



(i) Speeding up when $0 < t < 2$, $4 < t < 6$, $8 < t < 10$; slowing down when $2 < t < 4$, $6 < t < 8$

5. (a) Speeding up when $0 < t < 1$ or $2 < t < 3$; slowing down when $1 < t < 2$
(b) Speeding up when $1 < t < 2$ or $3 < t < 4$; slowing down when $0 < t < 1$ or $2 < t < 3$

7. (a) 4.9 m/s; -14.7 m/s (b) After 2.5 s (c) $32\frac{5}{16}$ m
(d) ≈ 5.08 s (e) ≈ -25.3 m/s

9. (a) 7.56 m/s (b) 6.24 m/s; -6.24 m/s

11. (a) 30 mm^2/mm; the rate at which the area is increasing with respect to side length as x reaches 15 mm
(b) $\Delta A \approx 2x\,\Delta x$

13. (a) (i) 5π (ii) 4.5π (iii) 4.1π
(b) 4π (c) $\Delta A \approx 2\pi r\,\Delta r$

15. (a) 8π ft^2/ft (b) 16π ft^2/ft (c) 24π ft^2/ft
The rate increases as the radius increases.

17. (a) 6 kg/m (b) 12 kg/m (c) 18 kg/m
At the right end; at the left end

19. (a) 4.75 A (b) 5 A; $t = \frac{2}{3}$ s

23. (a) $dV/dP = -C/P^2$ (b) At the beginning

25. $400(3^t)\ln 3$; ≈ 6850 bacteria/h

27. (a) 16 million/year; 78.5 million/year
(b) $P(t) = at^3 + bt^2 + ct + d$, where $a \approx 0.00129371$, $b \approx -7.061422$, $c \approx 12{,}822.979$, $d \approx -7{,}743{,}770$
(c) $P'(t) = 3at^2 + 2bt + c$
(d) 14.48 million/year; 75.29 million/year (smaller)
(e) 81.62 million/year

29. (a) 0.926 cm/s; 0.694 cm/s; 0
(b) 0; -92.6 (cm/s)/cm; -185.2 (cm/s)/cm
(c) At the center; at the edge

31. (a) $C'(x) = 12 - 0.2x + 0.0015x^2$
(b) \$32/yard; the cost of producing the 201st yard
(c) \$32.20

33. (a) $[xp'(x) - p(x)]/x^2$; the average productivity increases as new workers are added.

35. -0.2436 K/min

37. (a) 0 and 0 (b) $C = 0$
(c) (0, 0), (500, 50); it is possible for the species to coexist.

EXERCISES 3.8 ■ PAGE 242

1. About 235

3. (a) $100(4.2)^t$ (b) ≈ 7409 (c) $\approx 10{,}632$ bacteria/h
(d) $(\ln 100)/(\ln 4.2) \approx 3.2$ h

5. (a) 1508 million, 1871 million (b) 2161 million
(c) 3972 million; wars in the first half of century, increased life expectancy in second half

7. (a) $Ce^{-0.0005t}$ (b) $-2000\ln 0.9 \approx 211$ s

9. (a) $100 \times 2^{-t/30}$ mg (b) ≈ 9.92 mg (c) ≈ 199.3 years

11. ≈ 2500 years **13.** (a) $\approx 137°$F (b) ≈ 116 min

15. (a) $13.\overline{3}°$C (b) ≈ 67.74 min

17. (a) ≈ 64.5 kPa (b) ≈ 39.9 kPa

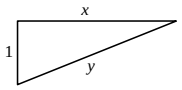
19. (a) (i) \$3828.84 (ii) \$3840.25 (iii) \$3850.08
(iv) \$3851.61 (v) \$3852.01 (vi) \$3852.08
(b) $dA/dt = 0.05A$, $A(0) = 3000$

EXERCISES 3.9 ■ PAGE 248

1. $dV/dt = 3x^2\,dx/dt$ **3.** 48 cm^2/s

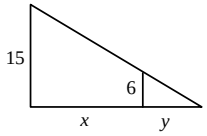
5. $3/(25\pi)$ m/min **7.** (a) 1 (b) 25 **9.** -18

11. (a) The plane's altitude is 1 mi and its speed is 500 mi/h.
(b) The rate at which the distance from the plane to the station is increasing when the plane is 2 mi from the station
(c)



(d) $y^2 = x^2 + 1$
(e) $250\sqrt{3}$ mi/h

13. (a) The height of the pole (15 ft), the height of the man (6 ft), and the speed of the man (5 ft/s)
(b) The rate at which the tip of the man's shadow is moving when he is 40 ft from the pole
(c)



(d) $\frac{15}{6} = \frac{x+y}{y}$ (e) $\frac{25}{3}$ ft/s

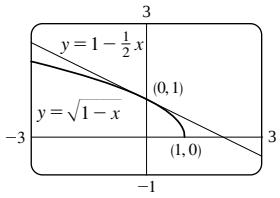
15. 65 mi/h **17.** $837/\sqrt{8674} \approx 8.99$ ft/s

19. -1.6 cm/min **21.** $\frac{720}{13} \approx 55.4$ km/h

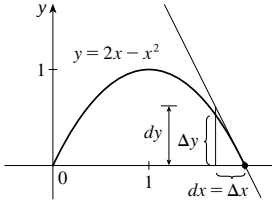
23. $(10{,}000 + 800{,}000\pi/9) \approx 2.89 \times 10^5$ cm^3/min
25. $\frac{10}{3}$ cm/min **27.** $6/(5\pi) \approx 0.38$ ft/min **29.** 0.3 m^2/s
31. 5 m **33.** 80 cm^3/min **35.** $\frac{107}{810} \approx 0.132\ \Omega$/s
37. 0.396 m/min **39.** (a) 360 ft/s (b) 0.096 rad/s
41. $\frac{10}{9}\pi$ km/min **43.** $1650/\sqrt{31} \approx 296$ km/h
45. $\frac{7}{4}\sqrt{15} \approx 6.78$ m/s

EXERCISES 3.10 ■ PAGE 255

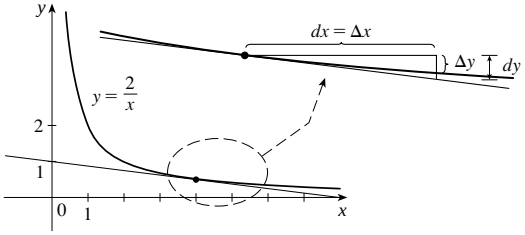
1. $L(x) = -10x - 6$ **3.** $L(x) = \frac{1}{4}x + 1$
5. $\sqrt{1-x} \approx 1 - \frac{1}{2}x$; $\sqrt{0.9} \approx 0.95$, $\sqrt{0.99} \approx 0.995$



7. $-0.383 < x < 0.516$ **9.** $-0.368 < x < 0.677$
11. (a) $dy = 2x(x\cos 2x + \sin 2x)\,dx$ (b) $dy = \dfrac{t}{1+t^2}\,dt$
13. (a) $dy = \dfrac{\sec^2\sqrt{t}}{2\sqrt{t}}\,dt$ (b) $dy = \dfrac{-4v}{(1+v^2)^2}\,dv$
15. (a) $dy = \frac{1}{10}e^{x/10}\,dx$ (b) 0.01
17. (a) $dy = \dfrac{x}{\sqrt{3+x^2}}\,dx$ (b) -0.05
19. $\Delta y = 0.64$, $dy = 0.8$



21. $\Delta y = -0.1$, $dy = -0.125$



23. 15.968 **25.** $10.00\overline{3}$ **27.** $1 - \pi/90 \approx 0.965$
33. (a) 270 cm^3, 0.01, 1% (b) 36 cm^2, $0.00\overline{6}$, $0.\overline{6}$%
35. (a) $84/\pi \approx 27$ cm^2; $\frac{1}{84} \approx 0.012 = 1.2\%$
(b) $1764/\pi^2 \approx 179$ cm^3; $\frac{1}{56} \approx 0.018 = 1.8\%$
37. (a) $2\pi rh\,\Delta r$ (b) $\pi(\Delta r)^2 h$
43. (a) 4.8, 5.2 (b) Too large

EXERCISES 3.11 ■ PAGE 262

1. (a) 0 (b) 1 **3.** (a) $\frac{3}{4}$ (b) $\frac{1}{2}(e^2 - e^{-2}) \approx 3.62686$
5. (a) 1 (b) 0
21. sech $x = \frac{3}{5}$, sinh $x = \frac{4}{3}$, csch $x = \frac{3}{4}$, tanh $x = \frac{4}{5}$, coth $x = \frac{5}{4}$
23. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1
(g) ∞ (h) $-\infty$ (i) 0
31. $f'(x) = x\cosh x$ **33.** $h'(x) = \tanh x$
35. $y' = 3e^{\cosh 3x}\sinh 3x$ **37.** $f'(t) = -2e^t\operatorname{sech}^2(e^t)\tanh(e^t)$
39. $G'(x) = \dfrac{-2\sinh x}{(1+\cosh x)^2}$ **41.** $y' = \dfrac{1}{2\sqrt{x(x-1)}}$
43. $y' = \sinh^{-1}(x/3)$ **45.** $y' = -\csc x$
51. (a) 0.3572 (b) 70.34°
53. (a) 164.50 m (b) 120 m; 164.13 m
55. (b) $y = 2\sinh 3x - 4\cosh 3x$
57. $\left(\ln(1+\sqrt{2}), \sqrt{2}\right)$

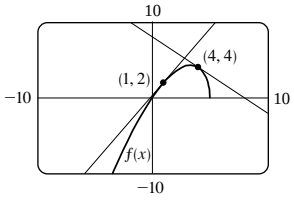
CHAPTER 3 REVIEW ■ PAGE 264

True-False Quiz

1. True **3.** True **5.** False **7.** False **9.** True
11. True **13.** True **15.** True

Exercises

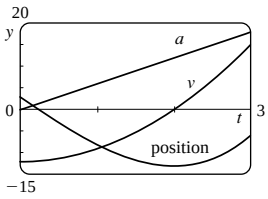
1. $4x^7(x+1)^3(3x+2)$ **3.** $\frac{3}{2}\sqrt{x} - \dfrac{1}{2\sqrt{x}} - \dfrac{1}{\sqrt{x^3}}$
5. $x(\pi x\cos\pi x + 2\sin\pi x)$
7. $\dfrac{8t^3}{(t^4+1)^2}$ **9.** $\dfrac{1+\ln x}{x\ln x}$ **11.** $\dfrac{\cos\sqrt{x} - \sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}$
13. $-\dfrac{e^{1/x}(1+2x)}{x^4}$ **15.** $\dfrac{2xy - \cos y}{1 - x\sin y - x^2}$
17. $\dfrac{1}{2\sqrt{\arctan x}\,(1+x^2)}$ **19.** $\dfrac{1-t^2}{(1+t^2)^2}\sec^2\left(\dfrac{t}{1+t^2}\right)$
21. $3^{x\ln x}(\ln 3)(1+\ln x)$ **23.** $-(x-1)^{-2}$
25. $\dfrac{2x - y\cos(xy)}{x\cos(xy) + 1}$ **27.** $\dfrac{2}{(1+2x)\ln 5}$
29. $\cot x - \sin x\cos x$ **31.** $\dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$
33. $5\sec 5x$ **35.** $-6x\csc^2(3x^2+5)$
37. $\cos\left(\tan\sqrt{1+x^3}\right)\left(\sec^2\sqrt{1+x^3}\right)\dfrac{3x^2}{2\sqrt{1+x^3}}$
39. $2\cos\theta\tan(\sin\theta)\sec^2(\sin\theta)$
41. $\dfrac{(x-2)^4(3x^2-55x-52)}{2\sqrt{x+1}\,(x+3)^8}$ **43.** $2x^2\cosh(x^2) + \sinh(x^2)$
45. $3\tanh 3x$ **47.** $\dfrac{\cosh x}{\sqrt{\sinh^2 x - 1}}$
49. $\dfrac{-3\sin\left(e^{\sqrt{\tan 3x}}\right)e^{\sqrt{\tan 3x}}\sec^2(3x)}{2\sqrt{\tan 3x}}$ **51.** $-\frac{4}{27}$
53. $-5x^4/y^{11}$ **57.** $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$
59. $y = 2x + 1$ **61.** $y = -x + 2$; $y = x + 2$
63. (a) $\dfrac{10 - 3x}{2\sqrt{5-x}}$ (b) $y = \frac{7}{4}x + \frac{1}{4}$, $y = -x + 8$
(c)



65. $(\pi/4, \sqrt{2}), (5\pi/4, -\sqrt{2})$ **69.** (a) 2 (b) 44
71. $2xg(x) + x^2g'(x)$ **73.** $2g(x)g'(x)$

75. $g'(e^x)e^x$ **77.** $g'(x)/g(x)$ **79.** $\dfrac{f'(x)[g(x)]^2 + g'(x)\,[f(x)]^2}{[f(x) + g(x)]^2}$

81. $f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4)$
83. $(-3, 0)$ **85.** $y = -\frac{2}{3}x^2 + \frac{14}{3}x$
87. $v(t) = -Ae^{-ct}[c\cos(\omega t + \delta) + \omega \sin(\omega t + \delta)]$,
$a(t) = Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega \sin(\omega t + \delta)]$
89. (a) $v(t) = 3t^2 - 12$; $a(t) = 6t$ (b) $t > 2$; $0 \leqslant t < 2$
(c) 23 (d) (e) $t > 2$; $0 < t < 2$



91. 4 kg/m
93. (a) $200(3.24)^t$ (b) $\approx$22,040
(c) $\approx$25,910 bacteria/h (d) $(\ln 50)/(\ln 3.24) \approx 3.33$ h
95. (a) C_0e^{-kt} (b) $\approx$100 h **97.** $\frac{4}{3}$ cm^2/min
99. 13 ft/s **101.** 400 ft/h
103. (a) $L(x) = 1 + x$; $\sqrt[3]{1 + 3x} \approx 1 + x$; $\sqrt[3]{1.03} \approx 1.01$
(b) $-0.235 < x < 0.401$
105. $12 + \frac{3}{2}\pi \approx 16.7$ cm^2 **107.** $\frac{1}{32}$ **109.** $\frac{1}{4}$ **111.** $\frac{1}{8}x^2$

PROBLEMS PLUS ■ PAGE 269

1. $\left(\pm\sqrt{3}/2, \frac{1}{4}\right)$ **5.** $3\sqrt{2}$ **11.** $\left(0, \frac{5}{4}\right)$
13. (a) $4\pi\sqrt{3}/\sqrt{11}$ rad/s (b) $40\left(\cos\theta + \sqrt{8 + \cos^2\theta}\right)$ cm
(c) $-480\pi \sin\theta\left(1 + \cos\theta/\sqrt{8 + \cos^2\theta}\right)$ cm/s
17. $x_T \in (3, \infty)$, $y_T \in (2, \infty)$, $x_N \in \left(0, \frac{5}{3}\right)$, $y_N \in \left(-\frac{5}{2}, 0\right)$
19. (b) (i) 53° (or 127°) (ii) 63° (or 117°)
21. R approaches the midpoint of the radius AO.
23. $-\sin a$ **25.** $2\sqrt{e}$ **29.** $(1, -2), (-1, 0)$
31. $\sqrt{29}/58$ **33.** $2 + \frac{375}{128}\pi \approx 11.204$ cm^3/min

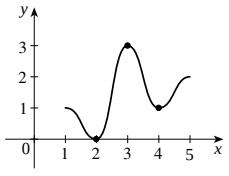
CHAPTER 4

EXERCISES 4.1 ■ PAGE 280

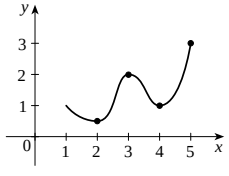
Abbreviations: abs, absolute; loc, local; max, maximum; min, minimum

1. Abs min: smallest function value on the entire domain of the function; loc min at c: smallest function value when x is near c
3. Abs max at s, abs min at r, loc max at c, loc min at b and r, neither a max nor a min at a and d
5. Abs max $f(4) = 5$, loc max $f(4) = 5$ and $f(6) = 4$, loc min $f(2) = 2$ and $f(1) = f(5) = 3$

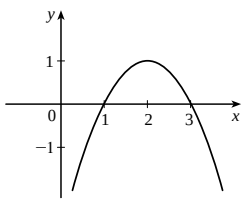
7.



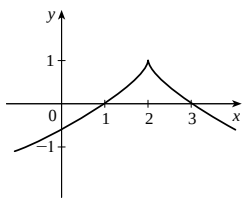
9.



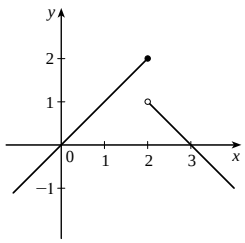
11. (a)



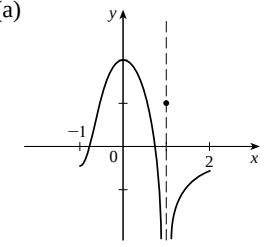
(b)



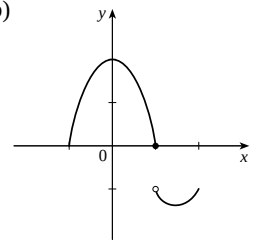
(c)



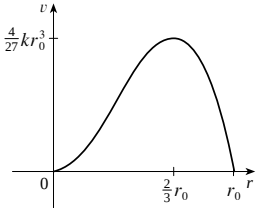
13. (a)



(b)



15. Abs max $f(3) = 4$ **17.** Abs max $f(1) = 1$
19. Abs min $f(0) = 0$
21. Abs max $f(\pi/2) = 1$; abs min $f(-\pi/2) = -1$
23. Abs max $f(2) = \ln 2$ **25.** Abs max $f(0) = 1$
27. Abs max $f(3) = 2$ **29.** $\frac{1}{3}$ **31.** $-2, 3$ **33.** 0
35. 0, 2 **37.** $0, \frac{4}{9}$ **39.** $0, \frac{8}{7}, 4$ **41.** $n\pi$ (n an integer)
43. $0, \frac{2}{3}$ **45.** 10 **47.** $f(2) = 16$, $f(5) = 7$
49. $f(-1) = 8$, $f(2) = -19$ **51.** $f(-2) = 33$, $f(2) = -31$
53. $f(0.2) = 5.2$, $f(1) = 2$ **55.** $f\left(\sqrt{2}\right) = 2$, $f(-1) = -\sqrt{3}$
57. $f(\pi/6) = \frac{3}{2}\sqrt{3}$, $f(\pi/2) = 0$
59. $f(2) = 2/\sqrt{e}$, $f(-1) = -1/\sqrt[8]{e}$
61. $f(1) = \ln 3$, $f\left(-\frac{1}{2}\right) = \ln\frac{3}{4}$
63. $f\left(\dfrac{a}{a + b}\right) = \dfrac{a^a b^b}{(a + b)^{a+b}}$
65. (a) 2.19, 1.81 (b) $\frac{6}{25}\sqrt{\frac{3}{5}} + 2$, $-\frac{6}{25}\sqrt{\frac{3}{5}} + 2$
67. (a) 0.32, 0.00 (b) $\frac{3}{16}\sqrt{3}$, 0 **69.** $\approx$3.9665°C
71. Cheapest, $t \approx 0.855$ (June 1994);
most expensive, $t \approx 4.618$ (March 1998)
73. (a) $r = \frac{2}{3}r_0$ (b) $v = \frac{4}{27}kr_0^3$
(c)



EXERCISES 4.2 ■ PAGE 288

1. 2 **3.** $\frac{9}{4}$ **5.** f is not differentiable on $(-1, 1)$
7. 0.3, 3, 6.3 **9.** 1 **11.** $3/\ln 4$ **13.** 1
15. f is not continous at 3 **23.** 16 **25.** No **31.** No

EXERCISES 4.3 ■ PAGE 297

Abbreviations: inc, increasing; dec, decreasing; CD, concave downward; CU, concave upward; HA, horizontal asymptote; VA, vertical asymptote; IP, inflection point(s)

1. (a) (1, 3), (4, 6) (b) (0, 1), (3, 4) (c) (0, 2)
(d) (2, 4), (4, 6) (e) (2, 3)

3. (a) I/D Test (b) Concavity Test
(c) Find points at which the concavity changes.

5. (a) Inc on (1, 5); dec on (0, 1) and (5, 6)
(b) Loc max at $x = 5$, loc min at $x = 1$

7. (a) 3, 5 (b) 2, 4, 6 (c) 1, 7

9. (a) Inc on $(-\infty, -3)$, $(2, \infty)$; dec on $(-3, 2)$
(b) Loc max $f(-3) = 81$; loc min $f(2) = -44$
(c) CU on $(-\frac{1}{2}, \infty)$; CD on $(-\infty, -\frac{1}{2})$; IP $(-\frac{1}{2}, \frac{37}{2})$

11. (a) Inc on $(-1, 0)$, $(1, \infty)$; dec on $(-\infty, -1)$, $(0, 1)$
(b) Loc max $f(0) = 3$; loc min $f(\pm 1) = 2$
(c) CU on $(-\infty, -\sqrt{3}/3)$, $(\sqrt{3}/3, \infty)$;
CD on $(-\sqrt{3}/3, \sqrt{3}/3)$; IP $(\pm\sqrt{3}/3, \frac{22}{9})$

13. (a) Inc on $(0, \pi/4)$, $(5\pi/4, 2\pi)$; dec on $(\pi/4, 5\pi/4)$
(b) Loc max $f(\pi/4) = \sqrt{2}$; loc min $f(5\pi/4) = -\sqrt{2}$
(c) CU on $(3\pi/4, 7\pi/4)$; CD on $(0, 3\pi/4)$, $(7\pi/4, 2\pi)$;
IP $(3\pi/4, 0)$, $(7\pi/4, 0)$

15. (a) Inc on $(-\frac{1}{3}\ln 2, \infty)$; dec on $(-\infty, -\frac{1}{3}\ln 2)$
(b) Loc min $f(-\frac{1}{3}\ln 2) = 2^{-2/3} + 2^{1/3}$ (c) CU on $(-\infty, \infty)$

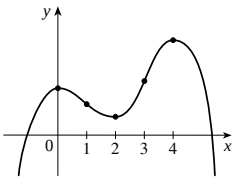
17. (a) Inc on $(1, \infty)$; dec on $(0, 1)$ (b) Loc min $f(1) = 0$
(c) CU on $(0, \infty)$; No IP

19. Loc max $f(1) = 2$; loc min $f(0) = 1$

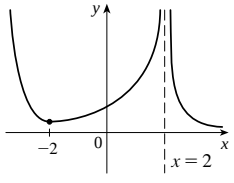
21. Loc min $f(\frac{1}{16}) = -\frac{1}{4}$

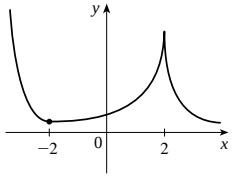
23. (a) f has a local maximum at 2.
(b) f has a horizontal tangent at 6.

25.

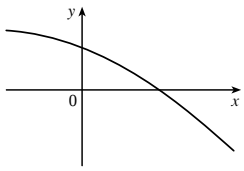


27.

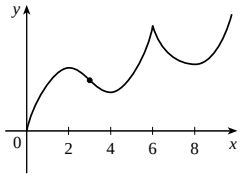




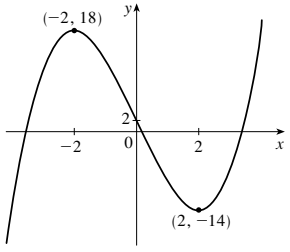
29.



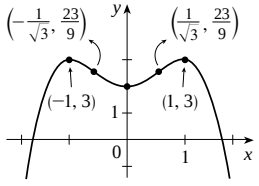
31. (a) Inc on $(0, 2)$, $(4, 6)$, $(8, \infty)$; dec on $(2, 4)$, $(6, 8)$
(b) Loc max at $x = 2, 6$;
loc min at $x = 4, 8$
(c) CU on $(3, 6)$, $(6, \infty)$;
CD on $(0, 3)$
(d) 3
(e) See graph at right.



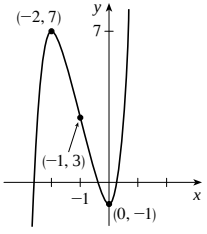
33. (a) Inc on $(-\infty, -2)$, $(2, \infty)$; dec on $(-2, 2)$
(b) Loc max $f(-2) = 18$; loc min $f(2) = -14$
(c) CU on $(0, \infty)$, CD on $(-\infty, 0)$; IP $(0, 2)$
(d)



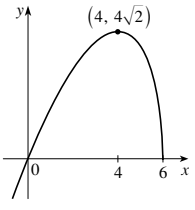
35. (a) Inc on $(-\infty, -1)$, $(0, 1)$;
dec on $(-1, 0)$, $(1, \infty)$
(b) Loc max $f(-1) = 3$, $f(1) = 3$;
loc min $f(0) = 2$
(c) CU on $(-1/\sqrt{3}, 1/\sqrt{3})$;
CD on $(-\infty, -1/\sqrt{3})$, $(1/\sqrt{3}, \infty)$;
IP $(\pm 1/\sqrt{3}, \frac{23}{9})$
(d) See graph at right.



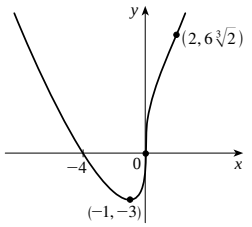
37. (a) Inc on $(-\infty, -2)$, $(0, \infty)$;
dec on $(-2, 0)$
(b) Loc max $h(-2) = 7$;
loc min $h(0) = -1$
(c) CU on $(-1, \infty)$;
CD on $(-\infty, -1)$; IP $(-1, 3)$
(d) See graph at right.



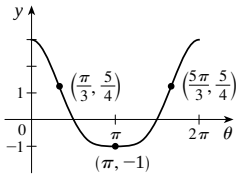
39. (a) Inc on $(-\infty, 4)$; dec on $(4, 6)$
(b) Loc max $f(4) = 4\sqrt{2}$
(c) CD on $(-\infty, 6)$; No IP
(d) See graph at right.



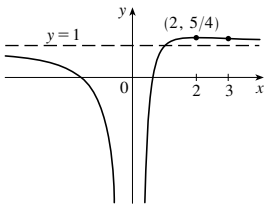
41. (a) Inc on $(-1, \infty)$;
dec on $(-\infty, -1)$
(b) Loc min $C(-1) = -3$
(c) CU on $(-\infty, 0)$, $(2, \infty)$;
CD on $(0, 2)$;
IP $(0, 0)$, $\left(2, 6\sqrt[3]{2}\right)$
(d) See graph at right.



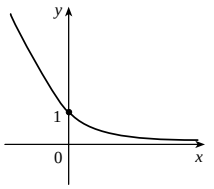
43. (a) Inc on $(\pi, 2\pi)$;
dec on $(0, \pi)$
(b) Loc min $f(\pi) = -1$
(c) CU on $(\pi/3, 5\pi/3)$;
CD on $(0, \pi/3)$, $(5\pi/3, 2\pi)$;
IP $\left(\pi/3, \frac{5}{4}\right)$, $\left(5\pi/3, \frac{5}{4}\right)$
(d) See graph at right.



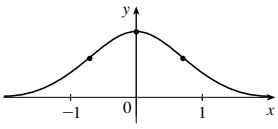
45. (a) VA $x = 0$; HA $y = 1$
(b) Inc on $(0, 2)$;
dec on $(-\infty, 0)$, $(2, \infty)$
(c) Loc max $f(2) = \frac{5}{4}$
(d) CU on $(3, \infty)$;
CD on $(-\infty, 0)$, $(0, 3)$; IP $\left(3, \frac{11}{9}\right)$
(e) See graph at right.



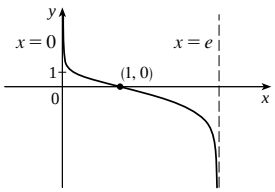
47. (a) HA $y = 0$
(b) Dec on $(-\infty, \infty)$
(c) None
(d) CU on $(-\infty, \infty)$
(e) See graph at right.



49. (a) HA $y = 0$
(b) Inc on $(-\infty, 0)$, dec on $(0, \infty)$
(c) Loc max $f(0) = 1$
(d) CU on $\left(-\infty, -1\sqrt{2}\right)$, $\left(1/\sqrt{2}, \infty\right)$;
CD on $\left(-1/\sqrt{2}, 1/\sqrt{2}\right)$;
IP $\left(\pm 1/\sqrt{2}, e^{-1/2}\right)$
(e) See graph at right.



51. (a) VA $x = 0$, $x = e$
(b) Dec on $(0, e)$
(c) None
(d) CU on $(0, 1)$; CD on $(1, e)$;
IP $(1, 0)$
(e) See graph at right.



53. $(3, \infty)$
55. (a) Loc and abs max $f(1) = \sqrt{2}$, no min
(b) $\frac{1}{4}\left(3 - \sqrt{17}\right)$
57. (b) CU on (0.94, 2.57), (3.71, 5.35);
CD on (0, 0.94), (2.57, 3.71), (5.35, 2π);
IP (0.94, 0.44), (2.57, −0.63), (3.71, −0.63), (5.35, 0.44)
59. CU on $(-\infty, -0.6)$, $(0.0, \infty)$; CD on $(-0.6, 0.0)$

61. (a) The rate of increase is initially very small, increases to a maximum at $t \approx 8$ h, then decreases toward 0.
(b) When $t = 8$ (c) CU on (0, 8); CD on (8, 18) (d) (8, 350)
63. $K(3) - K(2)$; CD
65. 28.57 min, when the rate of increase of drug level in the bloodstream is greatest; 85.71 min, when rate of decrease is greatest
67. $f(x) = \frac{1}{9}(2x^3 + 3x^2 - 12x + 7)$
69. (a) $a = 0$, $b = -1$ (b) $y = -x$ at (0, 0)

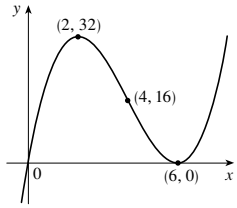
EXERCISES 4.4 ■ PAGE 307

1. (a) Indeterminate (b) 0 (c) 0
(d) ∞, $-\infty$, or does not exist (e) Indeterminate
3. (a) $-\infty$ (b) Indeterminate (c) ∞
5. $\frac{9}{4}$ **7.** 2 **9.** $-\frac{1}{3}$ **11.** $-\infty$ **13.** 2 **15.** $\frac{1}{4}$
17. 0 **19.** $-\infty$ **21.** $\frac{8}{5}$ **23.** 3 **25.** $\frac{1}{2}$ **27.** 1
29. 1 **31.** $1/\ln 3$ **33.** 0 **35.** $-1/\pi^2$ **37.** $\frac{1}{2}a(a-1)$
39. $\frac{1}{24}$ **41.** π **43.** 3 **45.** 0 **47.** $-2/\pi$ **49.** $\frac{1}{2}$
51. $\frac{1}{2}$ **53.** ∞ **55.** 1 **57.** e^{-2} **59.** $1/e$
61. 1 **63.** e^4 **65.** $1/\sqrt{e}$ **67.** e^2 **69.** $\frac{1}{4}$ **73.** 1
75. f has an absolute minimum for $c > 0$. As c increases, the minimum points get farther away from the origin.
81. $\frac{16}{9}a$ **83.** $\frac{1}{2}$ **85.** 56 **89.** (a) 0

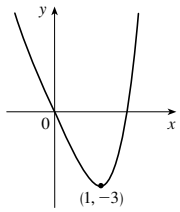
EXERCISES 4.5 ■ PAGE 317

Abbreviation: int, intercept

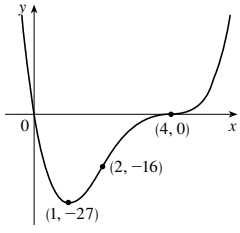
1. A. $\mathbb{R}$ B. y-int 0; x-int 0, 6
C. None D. None
E. Inc on $(-\infty, 2)$, $(6, \infty)$;
dec on $(2, 6)$
F. Loc max $f(2) = 32$;
loc min $f(6) = 0$
G. CU on $(4, \infty)$; CD on $(-\infty, 4)$;
IP $(4, 16)$
H. See graph at right.



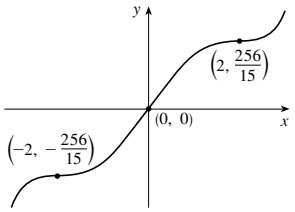
3. A. $\mathbb{R}$ B. y-int 0; x-int 0, $\sqrt[3]{4}$
C. None D. None
E. Inc on $(1, \infty)$; dec on $(-\infty, 1)$
F. Loc min $f(1) = -3$
G. CU on $(-\infty, \infty)$
H. See graph at right.



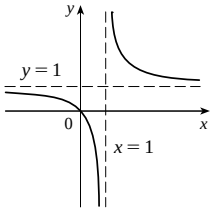
5. A. $\mathbb{R}$ B. y-int 0; x-int 0, 4
C. None D. None
E. Inc on $(1, \infty)$; dec on $(-\infty, 1)$
F. Loc min $f(1) = -27$
G. CU on $(-\infty, 2)$, $(4, \infty)$;
CD on $(2, 4)$;
IP $(2, -16)$, $(4, 0)$
H. See graph at right.



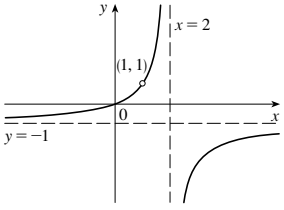
7. A. $\mathbb{R}$ B. y-int 0; x-int 0
C. About (0, 0) D. None
E. Inc on $(-\infty, \infty)$
F. None
G. CU on $(-2, 0)$, $(2, \infty)$;
CD on $(-\infty, -2)$, $(0, 2)$;
IP $\left(-2, -\frac{256}{15}\right)$, (0, 0), $\left(2, \frac{256}{15}\right)$
H. See graph at right.



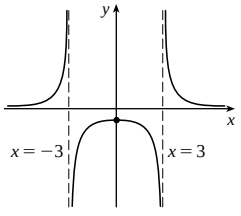
9. A. $\{x \mid x \neq 1\}$ B. y-int 0; x-int 0
C. None D. VA $x = 1$, HA $y = 1$
E. Dec on $(-\infty, 1)$, $(1, \infty)$
F. None
G. CU on $(1, \infty)$; CD on $(-\infty, 1)$
H. See graph at right.



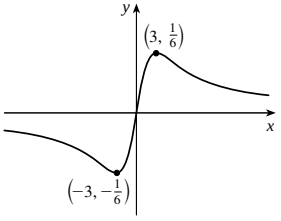
11. A. $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$
B. y-int 0; x-int 0 C. None
D. HA $y = -1$; VA $x = 2$
E. Inc on $(-\infty, 1)$, $(1, 2)$, $(2, \infty)$
F. None
G. CU on $(-\infty, 1)$, $(1, 2)$;
CD on $(2, \infty)$
H. See graph at right.



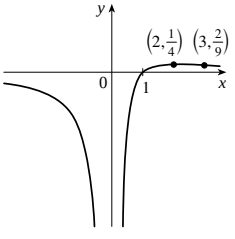
13. A. $\{x \mid x \neq \pm 3\}$ B. y-int $-\frac{1}{9}$
C. About y-axis D. VA $x = \pm 3$, HA $y = 0$
E. Inc on $(-\infty, -3)$, $(-3, 0)$;
dec on $(0, 3)$, $(3, \infty)$
F. Loc max $f(0) = -\frac{1}{9}$
G. CU on $(-\infty, -3)$, $(3, \infty)$;
CD on $(-3, 3)$
H. See graph at right.



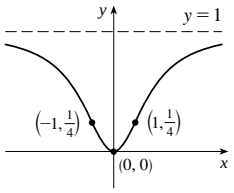
15. A. $\mathbb{R}$ B. y-int 0; x-int 0
C. About (0, 0) D. HA $y = 0$
E. Inc on $(-3, 3)$;
dec on $(-\infty, -3)$, $(3, \infty)$
F. Loc min $f(-3) = -\frac{1}{6}$;
loc max $f(3) = \frac{1}{6}$;
G. CU on $\left(-3\sqrt{3}, 0\right)$, $\left(3\sqrt{3}, \infty\right)$;
CD on $\left(-\infty, -3\sqrt{3}\right)$, $\left(0, 3\sqrt{3}\right)$;
IP (0, 0), $\left(\pm 3\sqrt{3}, \pm\sqrt{3}/12\right)$
H. See graph at right.



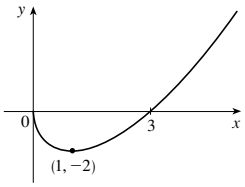
17. A. $(-\infty, 0) \cup (0, \infty)$ B. x-int 1
C. None D. HA $y = 0$; VA $x = 0$
E. Inc on $(0, 2)$;
dec on $(-\infty, 0)$, $(2, \infty)$
F. Loc max $f(2) = \frac{1}{4}$
G. CU on $(3, \infty)$;
CD on $(-\infty, 0)$, $(0, 3)$; IP $\left(3, \frac{2}{9}\right)$
H. See graph at right.



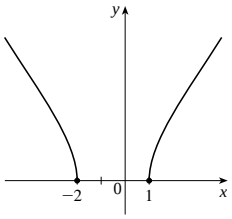
19. A. $\mathbb{R}$ B. y-int 0; x-int 0
C. About y-axis D. HA $y = 1$
E. Inc on $(0, \infty)$; dec on $(-\infty, 0)$
F. Loc min $f(0) = 0$
G. CU on $(-1, 1)$;
CD on $(-\infty, -1)$, $(1, \infty)$; IP $\left(\pm 1, \frac{1}{4}\right)$
H. See graph at right.



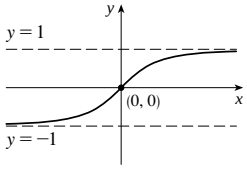
21. A. $[0, \infty)$ B. y-int 0; x-int 0, 3
C. None D. None
E. Inc on $(1, \infty)$; dec on $(0, 1)$
F. Loc min $f(1) = -2$
G. CU on $(0, \infty)$
H. See graph at right.



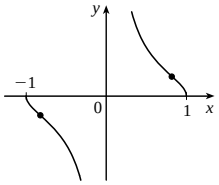
23. A. $(-\infty, -2] \cup [1, \infty)$
B. x-int -2, 1
C. None D. None
E. Inc on $(1, \infty)$; dec on $(-\infty, -2)$
F. None
G. CD on $(-\infty, -2)$, $(1, \infty)$
H. See graph at right.



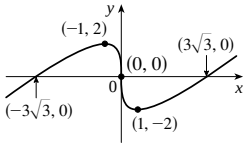
25. A. $\mathbb{R}$ B. y-int 0; x-int 0
C. About the origin
D. HA $y = \pm 1$
E. Inc on $(-\infty, \infty)$ F. None
G. CU on $(-\infty, 0)$;
CD on $(0, \infty)$; IP (0, 0)
H. See graph at right.



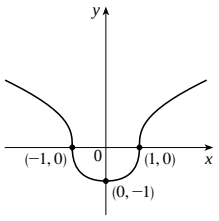
27. A. $\{x \mid |x| \leq 1, x \neq 0\} = [-1, 0) \cup (0, 1]$
B. x-int ± 1 C. About (0, 0)
D. VA $x = 0$
E. Dec on $(-1, 0)$, $(0, 1)$
F. None
G. CU on $\left(-1, -\sqrt{2/3}\right)$, $\left(0, \sqrt{2/3}\right)$;
CD on $\left(-\sqrt{2/3}, 0\right)$, $\left(\sqrt{2/3}, 1\right)$;
IP $\left(\pm\sqrt{2/3}, \pm 1/\sqrt{2}\right)$
H. See graph at right.



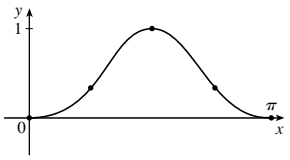
29. A. $\mathbb{R}$ B. y-int 0; x-int 0, $\pm 3\sqrt{3}$ C. About the origin
D. None E. Inc on $(-\infty, -1)$, $(1, \infty)$; dec on $(-1, 1)$
F. Loc max $f(-1) = 2$;
loc min $f(1) = -2$
G. CU on $(0, \infty)$; CD on $(-\infty, 0)$;
IP (0, 0)
H. See graph at right.



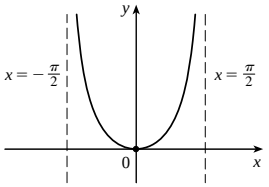
31. A. $\mathbb{R}$ B. y-int -1; x-int ± 1
C. About y-axis D. None
E. Inc on $(0, \infty)$; dec on $(-\infty, 0)$
F. Loc min $f(0) = -1$
G. CU on $(-1, 1)$;
CD on $(-\infty, -1)$, $(1, \infty)$;
IP $(\pm 1, 0)$
H. See graph at right.



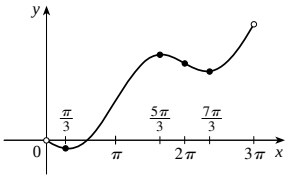
33. A. $\mathbb{R}$ B. y-int 0; x-int $n\pi$ (n an integer)
C. About (0, 0), period 2π
D. None
E–G answers for $0 \leqslant x \leqslant \pi$:
E. Inc on $(0, \pi/2)$; dec on $(\pi/2, \pi)$
F. Loc max $f(\pi/2) = 1$
G. Let $\alpha = \sin^{-1}\sqrt{2/3}$;
CU on $(0, \alpha)$, $(\pi - \alpha, \pi)$;
CD on $(\alpha, \pi - \alpha)$; IP at $x = 0, \pi, \alpha, \pi - \alpha$
H. See graph at right.



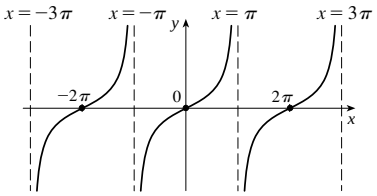
35. A. $(-\pi/2, \pi/2)$ B. y-int 0; x-int 0 C. About y-axis
D. VA $x = \pm\pi/2$
E. Inc on $(0, \pi/2)$;
dec on $(-\pi/2, 0)$
F. Loc min $f(0) = 0$
G. CU on $(-\pi/2, \pi/2)$
H. See graph at right.



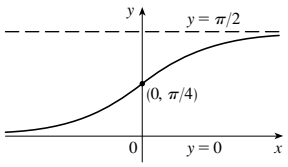
37. A. $(0, 3\pi)$ C. None D. None
E. Inc on $(\pi/3, 5\pi/3)$, $(7\pi/3, 3\pi)$;
dec on $(0, \pi/3)$, $(5\pi/3, 7\pi/3)$
F. Loc min $f(\pi/3) = (\pi/6) - \frac{1}{2}\sqrt{3}$, $f(7\pi/3) = (7\pi/6) - \frac{1}{2}\sqrt{3}$;
loc max $f(5\pi/3) = (5\pi/6) + \frac{1}{2}\sqrt{3}$
G. CU on $(0, \pi)$, $(2\pi, 3\pi)$;
CD on $(\pi, 2\pi)$;
IP $(\pi, \pi/2)$, $(2\pi, \pi)$
H. See graph at right.



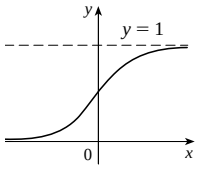
39. A. All reals except $(2n + 1)\pi$ (n an integer)
B. y-int 0; x-int $2n\pi$
C. About the origin, period 2π
D. VA $x = (2n + 1)\pi$
E. Inc on $((2n - 1)\pi, (2n + 1)\pi)$ F. None
G. CU on $(2n\pi, (2n + 1)\pi)$; CD on $((2n - 1)\pi, 2n\pi)$;
IP $(2n\pi, 0)$
H.



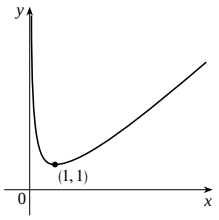
41. A. $\mathbb{R}$ B. y-int $\pi/4$
C. None
D. HA $y = 0$, $y = \pi/2$
E. Inc on $(-\infty, \infty)$ F. None
G. CU on $(-\infty, 0)$; CD on $(0, \infty)$;
IP $(0, \pi/4)$
H. See graph at right.



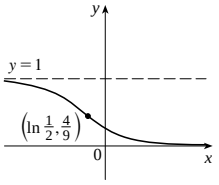
43. A. $\mathbb{R}$ B. y-int. $\frac{1}{2}$ C. None
D. HA $y = 0$, $y = 1$
E. Inc on $\mathbb{R}$ F. None
G. CU on $(-\infty, 0)$; CD on $(0, \infty)$;
IP $\left(0, \frac{1}{2}\right)$ H. See graph at right.



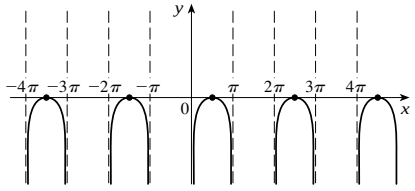
45. A. $(0, \infty)$ B. None
C. None D. VA $x = 0$
E. Inc on $(1, \infty)$; dec on $(0, 1)$
F. Loc min $f(1) = 1$
G. CU on $(0, \infty)$
H. See graph at right.



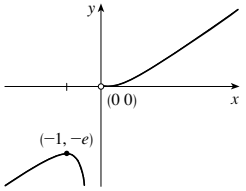
47. A. $\mathbb{R}$ B. y-int $\frac{1}{4}$ C. None
D. HA $y = 0$, $y = 1$
E. Dec on $\mathbb{R}$ F. None
G. CU on ; CD on $(-\infty, \ln\frac{1}{2})$;
IP $\left(\ln\frac{1}{2}, \frac{4}{9}\right)$
H. See graph at right.



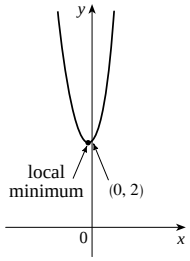
49. A. All x in $(2n\pi, (2n + 1)\pi)$ (n an integer)
B. x-int $\pi/2 + 2n\pi$ C. Period 2π D. VA $x = n\pi$
E. Inc on $(2n\pi, \pi/2 + 2n\pi)$; dec on $(\pi/2 + 2n\pi, (2n + 1)\pi)$
F. Loc max $f(\pi/2 + 2n\pi) = 0$ G. CD on $(2n\pi, (2n + 1)\pi)$
H.



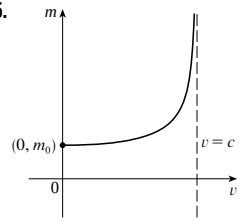
51. A. $(-\infty, 0) \cup (0, \infty)$
B. None C. None D. VA $x = 0$
E. Inc on $(-\infty, -1)$, $(0, \infty)$;
dec on $(-1, 0)$
F. Loc max $f(-1) = -e$
G. CU on $(0, \infty)$; CD on $(-\infty, 0)$
H. See graph at right.



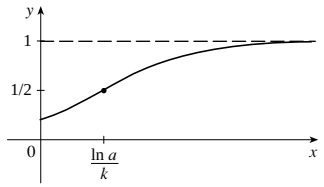
53. A. $\mathbb{R}$ B. y-int 2
C. None D. None
E. Inc on $\left(\frac{1}{5}\ln\frac{2}{3}, \infty\right)$; dec on $\left(-\infty, \frac{1}{5}\ln\frac{2}{3}\right)$
F. Loc min $f\left(\frac{1}{5}\ln\frac{2}{3}\right) = \left(\frac{2}{3}\right)^{3/5} + \left(\frac{2}{3}\right)^{-2/5}$
G. CU on $(-\infty, \infty)$
H. See graph at right.



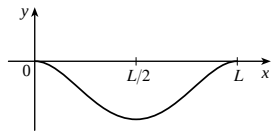
55.



57. (a) When $t = (\ln a)/k$ (b) When $t = (\ln a)/k$
(c)

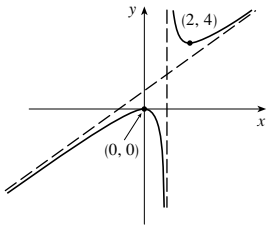


59.

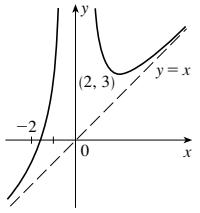


61. $y = x - 1$ **63.** $y = 2x - 2$

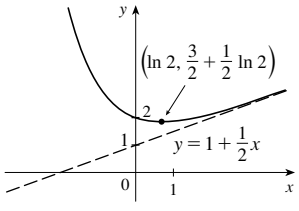
65. A. $(-\infty, 1) \cup (1, \infty)$
B. y-int 0; x-int 0 C. None
D. VA $x = 1$; SA $y = x + 1$
E. Inc on $(-\infty, 0)$, $(2, \infty)$;
dec on $(0, 1)$, $(1, 2)$
F. Loc max $f(0) = 0$;
loc min $f(2) = 4$
G. CU on $(1, \infty)$; CD on $(-\infty, 1)$
H. See graph at right.



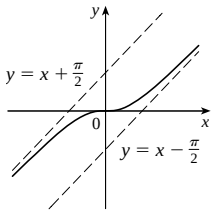
67. A. $(-\infty, 0) \cup (0, \infty)$
B. x-int $-\sqrt[3]{4}$ C. None
D. VA $x = 0$; SA $y = x$
E. Inc. on $(-\infty, 0)$, $(2, \infty)$;
dec on $(0, 2)$
F. Loc min $f(2) = 3$
G. CU on $(-\infty, 0)$, $(0, \infty)$
H. See graph at right.



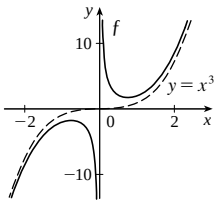
69. A. $\mathbb{R}$ B. y-int 2
C. None
D. SA $y = 1 + \frac{1}{2}x$
E. Inc. on $(\ln 2, \infty)$;
dec on $(-\infty, \ln 2)$
F. Loc min $f(\ln 2) = \frac{3}{2} + \frac{1}{2}\ln 2$
G. CU on $(-\infty, \infty)$
H. See graph at right.



71.

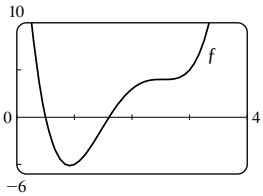


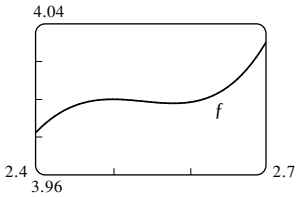
75. VA $x = 0$, asymptotic to $y = x^3$



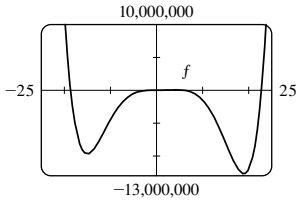
EXERCISES 4.6 ■ PAGE 324

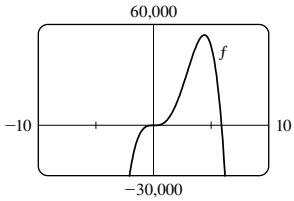
1. Inc on $(0.92, 2.5)$, $(2.58, \infty)$; dec on , $(2.5, 2.58)$;
loc max $f(2.5) = 4$; loc min $f(0.92) \approx -5.12$, $f(2.58) \approx 3.998$;
CU on $(-\infty, 1.46)$, $(2.54, \infty)$;
CD on $(1.46, 2.54)$; IP $(1.46, -1.40)$, $(2.54, 3.999)$



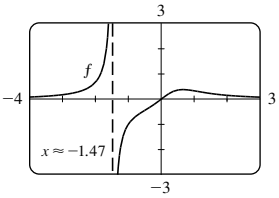


3. Inc on $(-15, 4.40)$, $(18.93, \infty)$;
dec on $(-\infty, -15)$, $(4.40, 18.93)$;
loc max $f(4.40) \approx 53{,}800$; loc min $f(-15) \approx -9{,}700{,}000$,
$f(18.93) \approx -12{,}700{,}000$; CU on $(-\infty, -11.34)$, $(0, 2.92)$,
$(15.08, \infty)$; CD on $(-11.34, 0)$, $(2.92, 15.08)$;
IP $(0, 0)$, $\approx (-11.34, -6{,}250{,}000)$, $(2.92, 31{,}800)$,
$(15.08, -8{,}150{,}000)$

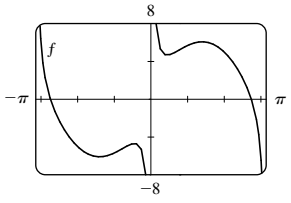




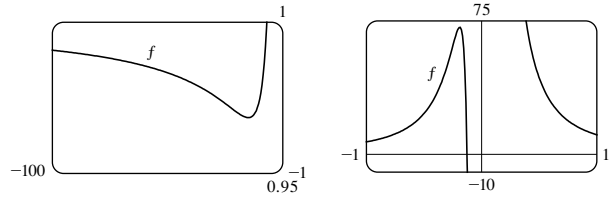
5. Inc on $(-\infty, -1.47)$, $(-1.47, 0.66)$; dec on $(0.66, \infty)$;
loc max $f(0.66) \approx 0.38$; CU on $(-\infty, -1.47)$, $(-0.49, 0)$,
$(1.10, \infty)$; CD on $(-1.47, -0.49)$, $(0, 1.10)$;
IP $(-0.49, -0.44)$, $(1.10, 0.31)$



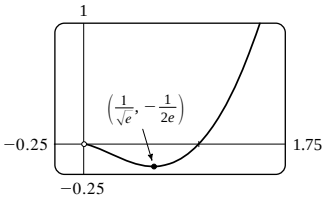
7. Inc on $(-1.40, -0.44)$, $(0.44, 1.40)$; dec on $(-\pi, -1.40)$, $(-0.44, 0)$, $(0, 0.44)$, $(1.40, \pi)$; loc max $f(-0.44) \approx -4.68$, $f(1.40) \approx 6.09$; loc min $f(-1.40) \approx -6.09$, $f(0.44) \approx 5.22$; CU on $(-\pi, -0.77)$, $(0, 0.77)$; CD on $(-0.77, 0)$, $(0.77, \pi)$; IP $(-0.77, -5.22)$, $(0.77, 5.22)$



9. Inc on $\left(-8 - \sqrt{61}, -8 + \sqrt{61}\right)$; dec on $\left(-\infty, -8 - \sqrt{61}\right)$, $\left(-8 + \sqrt{61}, 0\right)$, $(0, \infty)$; CU on $\left(-12 - \sqrt{138}, -12 + \sqrt{138}\right)$, $(0, \infty)$; CD on $\left(-\infty, -12 - \sqrt{138}\right)$, $\left(-12 + \sqrt{138}, 0\right)$



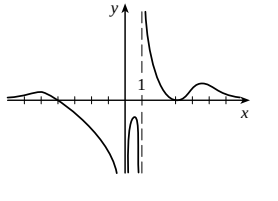
11. (a)

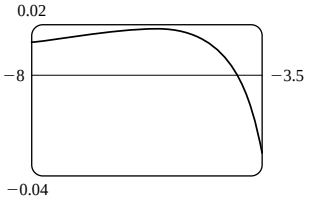


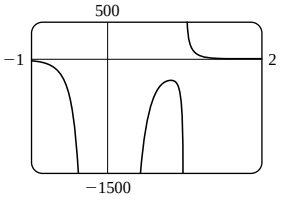
(b) $\lim_{x \to 0^+} f(x) = 0$

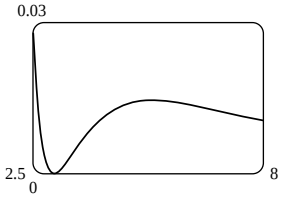
(c) Loc min $f\left(1/\sqrt{e}\right) = -1/(2e)$; CD on $(0, e^{-3/2})$; CU on $(e^{-3/2}, \infty)$

13. Loc max $f(-5.6) \approx 0.018$, $f(0.82) \approx -281.5$, $f(5.2) \approx 0.0145$; loc min $f(3) = 0$







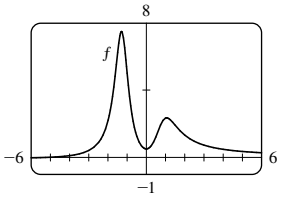


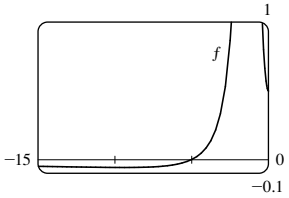
15. $f'(x) = -\dfrac{x(x+1)^2(x^3 + 18x^2 - 44x - 16)}{(x-2)^3(x-4)^5}$

$f''(x) = 2\dfrac{(x+1)(x^6 + 36x^5 + 6x^4 - 628x^3 + 684x^2 + 672x + 64)}{(x-2)^4(x-4)^6}$

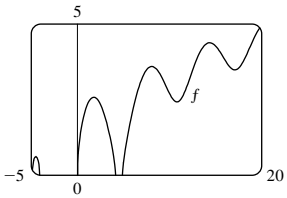
CU on $(-35.3, -5.0)$, $(-1, -0.5)$, $(-0.1, 2)$, $(2, 4)$, $(4, \infty)$;
CD on $(-\infty, -35.3)$, $(-5.0, -1)$, $(-0.5, -0.1)$;
IP $(-35.3, -0.015)$, $(-5.0, -0.005)$, $(-1, 0)$, $(-0.5, 0.00001)$, $(-0.1, 0.0000066)$

17. Inc on $(-9.41, -1.29)$, $(0, 1.05)$; dec on $(-\infty, -9.41)$, $(-1.29, 0)$, $(1.05, \infty)$; loc max $f(-1.29) \approx 7.49$, $f(1.05) \approx 2.35$; loc min $f(-9.41) \approx -0.056$, $f(0) = 0.5$; CU on $(-13.81, -1.55)$, $(-1.03, 0.60)$, $(1.48, \infty)$; CD on $(-\infty, -13.81)$, $(-1.55, -1.03)$, $(0.60, 1.48)$; IP $(-13.81, -0.05)$, $(-1.55, 5.64)$, $(-1.03, 5.39)$, $(0.60, 1.52)$, $(1.48, 1.93)$

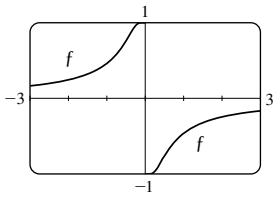




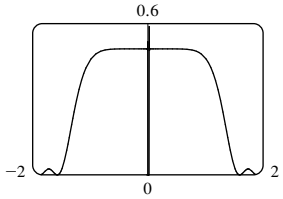
19. Inc on $(-4.91, -4.51)$, $(0, 1.77)$, $(4.91, 8.06)$, $(10.79, 14.34)$, $(17.08, 20)$;
dec on $(-4.51, -4.10)$, $(1.77, 4.10)$, $(8.06, 10.79)$, $(14.34, 17.08)$;
loc max $f(-4.51) \approx 0.62$, $f(1.77) \approx 2.58$, , $f(14.34) \approx 4.39$;
loc min $f(10.79) \approx 2.43$, $f(17.08) \approx 3.49$;
CU on $(9.60, 12.25)$, $(15.81, 18.65)$;
CD on $(-4.91, -4.10)$, $(0, 4.10)$, $(4.91, 9.60)$, $(12.25, 15.81)$, $(18.65, 20)$;
IP at $(9.60, 2.95)$, $(12.25, 3.27)$, $(15.81, 3.91)$, $(18.65, 4.20)$

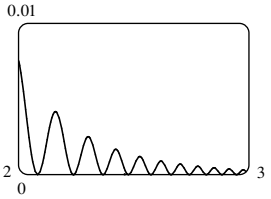


21. Inc on $(-\infty, 0)$, $(0, \infty)$;
CU on $(-\infty, -0.42)$, $(0, 0.42)$;
CD on $(-0.42, 0)$, $(0.42, \infty)$;
IP $(\mp 0.42, \pm 0.83)$

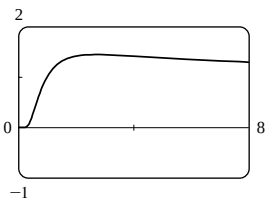


23.





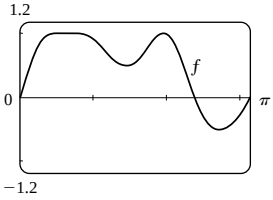
25. (a)

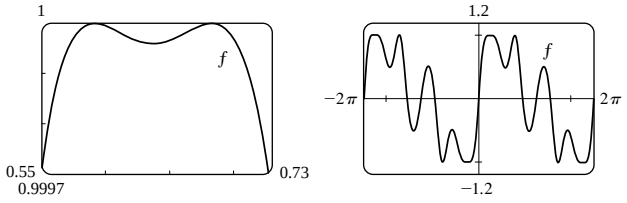


(b) $\lim_{x \to 0^+} x^{1/x} = 0$, $\lim_{x \to \infty} x^{1/x} = 1$

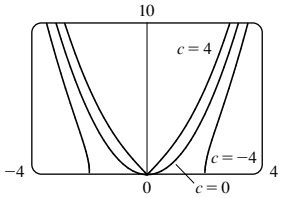
(c) Loc max $f(e) = e^{1/e}$ (d) IP at $x \approx 0.58, 4.37$

27. Max $f(0.59) \approx 1$, $f(0.68) \approx 1$, $f(1.96) \approx 1$; min $f(0.64) \approx 0.99996$, $f(1.46) \approx 0.49$, $f(2.73) \approx -0.51$; IP (0.61, 0.99998), (0.66, 0.99998), (1.17, 0.72), (1.75, 0.77), (2.28, 0.34)

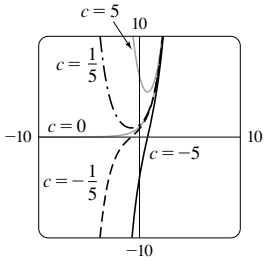




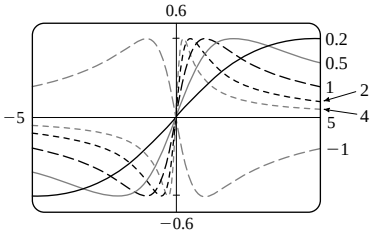
29. For $c \geqslant 0$, there is an absolute minimum at the origin. There are no other maximums or minimums. The more negative c becomes, the farther the two IPs move from the origin. $c = 0$ is a transitional value.



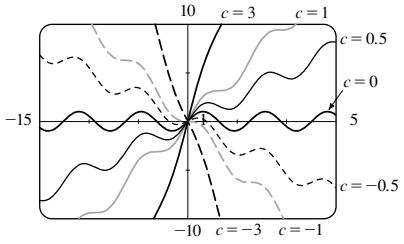
31. For $c < 0$, there is no extreme point and one IP, which decreases along the x-axis. For $c > 0$, there is no IP, and one minimum point.



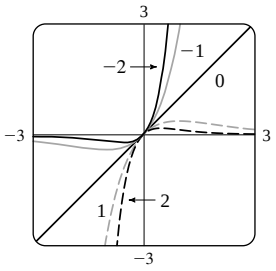
33. For $c > 0$, the maximum and minimum values are always $\pm\frac{1}{2}$, but the extreme points and IPs move closer to the y-axis as c increases. $c = 0$ is a transitional value: when c is replaced by $-c$, the curve is reflected in the x-axis.



35. For $|c| < 1$, the graph has loc max and min values; for $|c| \geqslant 1$ it does not. The function increases for $c \geqslant 1$ and decreases for $c \leqslant -1$. As c changes, the IPs move vertically but not horizontally.

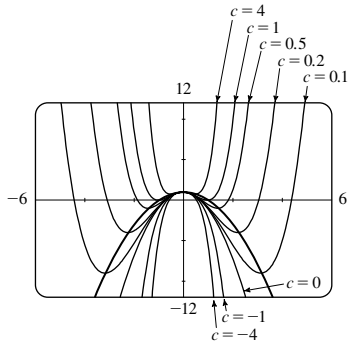


37.



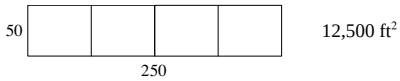
For $c > 0$, $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = -\infty$.
For $c < 0$, $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = 0$.
As $|c|$ increases, the max and min points and the IPs get closer to the origin.

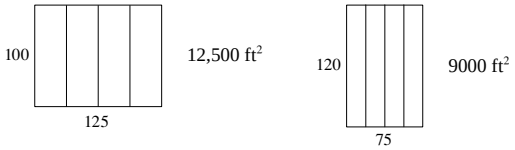
39. (a) Positive (b)



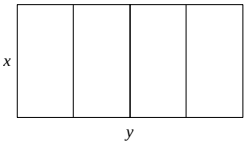
EXERCISES 4.7 ■ PAGE 331

1. (a) 11, 12 (b) 11.5, 11.5 **3.** 10, 10 **5.** $\frac{9}{4}$
7. 25 m by 25 m **9.** $N = 1$
11. (a)





(b)



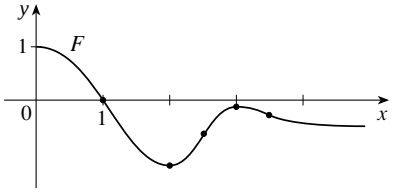
(c) $A = xy$ (d) $5x + 2y = 750$ (e) $A(x) = 375x - \frac{5}{2}x^2$
(f) 14,062.5 ft^2
13. 1000 ft by 1500 ft **15.** 4000 cm^3 **17.** \$191.28
19. $\left(-\frac{6}{5}, \frac{3}{5}\right)$ **21.** $\left(-\frac{1}{3}, \pm\frac{4}{3}\sqrt{2}\right)$ **23.** Square, side $\sqrt{2}r$
25. $L/2$, $\sqrt{3}\,L/4$ **27.** Base $\sqrt{3}r$, height $3r/2$
29. $4\pi r^3/(3\sqrt{3})$ **31.** $\pi r^2(1 + \sqrt{5})$ **33.** 24 cm, 36 cm
35. (a) Use all of the wire for the square
(b) $40\sqrt{3}/(9 + 4\sqrt{3})$ m for the square
37. Height = radius = $\sqrt[3]{V/\pi}$ cm
39. $V = 2\pi R^3/(9\sqrt{3})$ **43.** $E^2/(4r)$
45. (a) $\frac{3}{2}s^2 \csc\theta\,(\csc\theta - \sqrt{3}\cot\theta)$ (b) $\cos^{-1}(1/\sqrt{3}) \approx 55°$
(c) $6s[h + s/(2\sqrt{2})]$
47. Row directly to B **49.** $\approx$4.85 km east of the refinery
51. $10\sqrt[3]{3}/(1 + \sqrt[3]{3})$ ft from the stronger source
53. $(a^{2/3} + b^{2/3})^{3/2}$ **55.** $2\sqrt{6}$
57. (b) (i) \$342,491; \$342/unit; \$390/unit (ii) 400
(iii) \$320/unit
59. (a) $p(x) = 19 - \frac{1}{3000}x$ (b) \$9.50
61. (a) $p(x) = 550 - \frac{1}{10}x$ (b) \$175 (c) \$100
65. 9.35 m **69.** $x = 6$ in. **71.** $\pi/6$
73. At a distance $5 - 2\sqrt{5}$ from A **75.** $\frac{1}{2}(L + W)^2$
77. (a) About 5.1 km from B (b) C is close to B; C is close to D; $W/L = \sqrt{25 + x^2}/x$, where $x = |BC|$
(c) $\approx$1.07; no such value (d) $\sqrt{41}/4 \approx 1.6$

EXERCISES 4.8 ■ PAGE 342

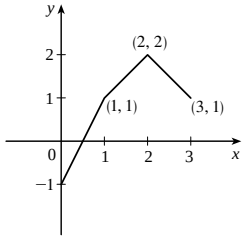
1. (a) $x_2 \approx 2.3$, $x_3 \approx 3$ (b) No
3. $\frac{9}{2}$ **5.** a, b, c **7.** 1.1785 **9.** -1.25 **11.** 1.82056420
13. 1.217562 **15.** -1.964636
17. -3.637958, -1.862365, 0.889470
19. 1.412391, 3.057104 **21.** 0, ± 0.902025
23. -1.93822883, -1.21997997, 1.13929375, 2.98984102
25. 0.76682579 **27.** 0.21916368, 1.08422462
29. (b) 31.622777
35. (a) -1.293227, -0.441731, 0.507854 (b) -2.0212
37. (1.520092, 2.306964) **39.** (0.410245, 0.347810)
41. 0.76286%

EXERCISES 4.9 ■ PAGE 348

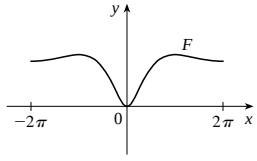
1. $F(x) = \frac{1}{2}x^2 - 3x + C$ **3.** $F(x) = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$
5. $F(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$ **7.** $F(x) = 5x^{7/5} + 40x^{1/5} + C$
9. $F(x) = \sqrt{2}x + C$ **11.** $F(x) = 2x^{3/2} - \frac{3}{2}x^{4/3} + C$
13. $F(x) = \begin{cases} \frac{1}{5}x - 2\ln|x| + C_1 & \text{if } x < 0 \\ \frac{1}{5}x - 2\ln|x| + C_2 & \text{if } x > 0 \end{cases}$
15. $G(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C$
17. $H(\theta) = -2\cos\theta - \tan\theta + C_n$ on $(n\pi - \pi/2, n\pi + \pi/2)$, n an integer
19. $F(x) = 5e^x - 3\sinh x + C$
21. $F(x) = \frac{1}{2}x^2 - \ln|x| - 1/x^2 + C$
23. $F(x) = x^5 - \frac{1}{3}x^6 + 4$ **25.** $f(x) = x^5 - x^4 + x^3 + Cx + D$
27. $\frac{3}{20}x^{8/3} + Cx + D$ **29.** $f(t) = -\sin t + Ct^2 + Dt + E$
31. $f(x) = x + 2x^{3/2} + 5$ **33.** $f(t) = 4\arctan t - \pi$
35. $2\sin t + \tan t + 4 - 2\sqrt{3}$
37. $\frac{3}{2}x^{2/3} - \frac{1}{2}$ if $x > 0$; $\frac{3}{2}x^{2/3} - \frac{5}{2}$ if $x < 0$
39. $-x^2 + 2x^3 - x^4 + 12x + 4$ **41.** $-\sin\theta - \cos\theta + 5\theta + 4$
43. $f(x) = 2x^2 + x^3 + 2x^4 + 2x + 3$ **45.** $x^2 - \cos x - \frac{1}{2}\pi x$
47. $-\ln x + (\ln 2)x - \ln 2$ **49.** 10 **51.** b
53.



55.



57.



59. $s(t) = 1 - \cos t - \sin t$
61. $s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + 3$
63. $s(t) = -10\sin t - 3\cos t + (6/\pi)t + 3$
65. (a) $s(t) = 450 - 4.9t^2$ (b) $\sqrt{450/4.9} \approx 9.58$ s
(c) $-9.8\sqrt{450/4.9} \approx -93.9$ m/s (d) About 9.09 s
69. 225 ft **71.** \$742.08 **73.** $\frac{130}{11} \approx 11.8$ s
75. $\frac{88}{15} \approx 5.87$ ft/s^2 **77.** 62,500 km/h^2 ≈ 4.82 m/s^2
79. (a) 22.9125 mi (b) 21.675 mi (c) 30 min 33 s
(d) 55.425 mi

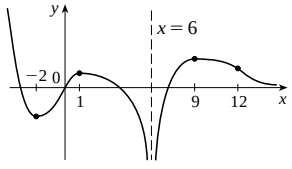
CHAPTER 4 REVIEW ■ PAGE 351

True-False Quiz

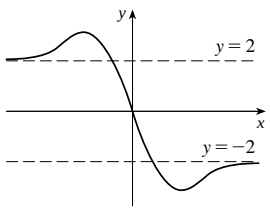
1. False **3.** False **5.** True **7.** False **9.** True
11. True **13.** False **15.** True **17.** True **19.** True

Exercises

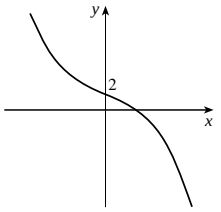
1. Abs max $f(4) = 5$, abs and loc min $f(3) = 1$
3. Abs max $f(2) = \frac{2}{5}$, abs and loc min $f\left(-\frac{1}{3}\right) = -\frac{9}{2}$
5. Abs and loc max $f(\pi/6) = \pi/6 + \sqrt{3}$,
abs min $f(-2) = -\pi - 2$, loc min $f(5\pi/6) = 5\pi/6 - \sqrt{3}$
7. 1 **9.** 8 **11.** 0 **13.** $\frac{1}{2}$
15.



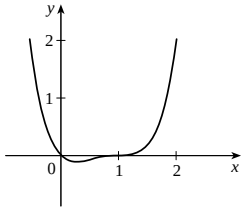
17.



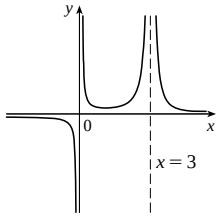
19. A. $\mathbb{R}$ B. y-int 2
C. None D. None
E. Dec on $(-\infty, \infty)$ F. None
G. CU on $(-\infty, 0)$;
CD on $(0, \infty)$; IP $(0, 2)$
H. See graph at right.



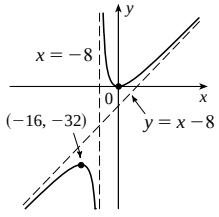
21. A. $\mathbb{R}$ B. y-int 0; x-int 0, 1
C. None D. None
E. Inc on $\left(\frac{1}{4}, \infty\right)$, dec on $\left(-\infty, \frac{1}{4}\right)$
F. Loc min $f\left(\frac{1}{4}\right) = -\frac{27}{256}$
G. CU on $\left(-\infty, \frac{1}{2}\right)$, $(1, \infty)$;
CD on $\left(\frac{1}{2}, 1\right)$; IP $\left(\frac{1}{2}, -\frac{1}{16}\right)$, $(1, 0)$
H. See graph at right.



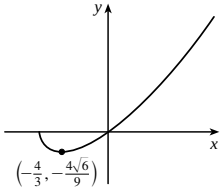
23. A. $\{x \mid x \neq 0, 3\}$
B. None C. None
D. HA $y = 0$; VA $x = 0$, $x = 3$
E. Inc on $(1, 3)$; dec on $(-\infty, 0)$, $(0, 1)$, $(3, \infty)$
F. Loc min $f(1) = \frac{1}{4}$
G. CU on $(0, 3)$, $(3, \infty)$; CD on $(-\infty, 0)$
H. See graph at right.



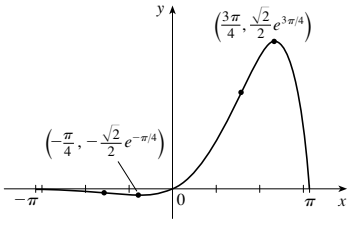
25. A. $\{x \mid x \neq -8\}$
B. y-int 0, x-int 0 C. None
D. VA $x = -8$; SA $y = x - 8$
E. Inc on $(-\infty, -16)$, $(0, \infty)$;
dec on $(-16, -8)$, $(-8, 0)$
F. Loc max $f(-16) = -32$;
loc min
G. CU on $(-8, \infty)$; CD on $(-\infty, -8)$
H. See graph at right.



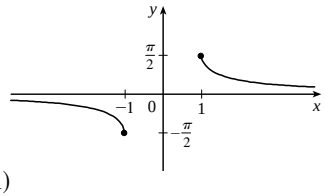
27. A. $[-2, \infty)$
B. y-int 0; x-int -2, 0
C. None D. None
E. Inc on $\left(-\frac{4}{3}, \infty\right)$, dec on $\left(-2, -\frac{4}{3}\right)$
F. Loc min $f\left(-\frac{4}{3}\right) = -\frac{4}{9}\sqrt{6}$
G. CU on $(-2, \infty)$
H. See graph at right.



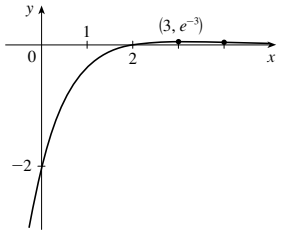
29. A. $[-\pi, \pi]$ B. y-int 0; x-int $-\pi$, 0, π
C. None D. None
E. Inc on $\left(-\pi/4, 3\pi/4\right)$; dec on $\left(-\pi, -\pi/4\right)$, $\left(3\pi/4, \pi\right)$
F. Loc max $f(3\pi/4) = \frac{1}{2}\sqrt{2}\,e^{3\pi/4}$, loc min $f(-\pi/4) = -\frac{1}{2}\sqrt{2}\,e^{3\pi/4}$
G. CU on $(-\pi/2, \pi/2)$; CD on $(-\pi, -\pi/2)$, $(\pi/2, \pi)$;
IP $(-\pi/2, -e^{-\pi/2})$, $(\pi/2, e^{\pi/2})$
H.



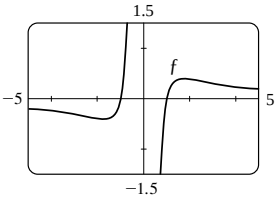
31. A. $\{x \mid |x| \geq 1\}$
B. None C. About (0, 0)
D. HA $y = 0$
E. Dec on $(-\infty, -1)$, $(1, \infty)$
F. None
G. CU on $(1, \infty)$; CD on $(-\infty, -1)$
H. See graph at right.



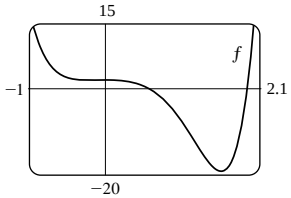
33. A. $\mathbb{R}$
B. y-int -2; x-int 2
C. None D. HA $y = 0$
E. Inc on $(-\infty, 3)$; dec on $(3, \infty)$
F. Loc max $f(3) = e^{-3}$
G. CU on $(4, \infty)$; CD on $(-\infty, 4)$;
IP $(4, 2e^{-4})$
H. See graph at right.

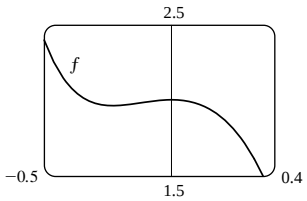


35. Inc on $\left(-\sqrt{3}, 0\right)$, $\left(0, \sqrt{3}\right)$;
dec on $\left(-\infty, -\sqrt{3}\right)$, $\left(\sqrt{3}, \infty\right)$;
loc max $f\left(\sqrt{3}\right) = \frac{2}{9}\sqrt{3}$,
loc min $f\left(-\sqrt{3}\right) = -\frac{2}{9}\sqrt{3}$;
CU on $\left(-\sqrt{6}, 0\right)$, $\left(\sqrt{6}, \infty\right)$;
CD on $\left(-\infty, -\sqrt{6}\right)$, $\left(0, \sqrt{6}\right)$;
IP $\left(\sqrt{6}, \frac{5}{36}\sqrt{6}\right)$, $\left(-\sqrt{6}, -\frac{5}{36}\sqrt{6}\right)$

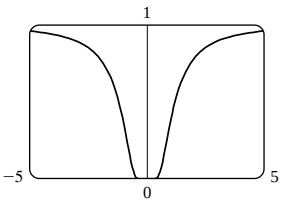


37. Inc on $(-0.23, 0)$, $(1.62, \infty)$; dec on $(-\infty, -0.23)$, $(0, 1.62)$;
loc max $f(0) = 2$; loc min $f(-0.23) \approx 1.96$, $f(1.62) \approx -19.2$;
CU on $(-\infty, -0.12)$, $(1.24, \infty)$;
CD on $(-0.12, 1.24)$; IP $(-0.12, 1.98)$, $(1.24, -12.1)$





39.



$(\pm 0.82, 0.22)$; $\left(\pm\sqrt{2/3}, e^{-3/2}\right)$

41. -2.96, -0.18, 3.01; -1.57, 1.57; -2.16, -0.75, 0.46, 2.21

43. For $C > -1$, f is periodic with period 2π and has local maxima at $2n\pi + \pi/2$, n an integer. For $C \leq -1$, f has no graph. For $-1 < C \leq 1$, f has vertical asymptotes. For $C > 1$, f is continuous on $\mathbb{R}$. As C increases, f moves upward and its oscillations become less pronounced.

49. (a) 0 (b) CU on $\mathbb{R}$ **53.** $3\sqrt{3}\,r^2$
55. $4/\sqrt{3}$ cm from D **57.** $L = C$ **59.** \$11.50
61. 1.297383 **63.** 1.16718557

65. $f(x) = \sin x - \sin^{-1}x + C$

67. $f(x) = \frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C$

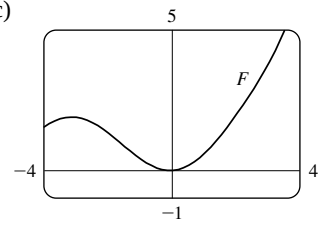
69. $f(t) = t^2 + 3\cos t + 2$

71. $f(x) = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + 1$

73. $s(t) = t^2 - \tan^{-1}t + 1$

75. (b) $0.1e^x - \cos x + 0.9$

(c)



77. No

79. (b) About 8.5 in. by 2 in. (c) $20/\sqrt{3}$ in., $20\sqrt{2/3}$ in.

83. (a) $20\sqrt{2} \approx 28$ ft

(b) $\dfrac{dI}{dt} = \dfrac{-480k(h-4)}{[(h-4)^2 + 1600]^{5/2}}$, where k is the constant of proportionality

PROBLEMS PLUS ■ PAGE 356

3. Abs max $f(-5) = e^{45}$, no abs min **7.** 24

9. $(-2, 4)$, $(2, -4)$ **13.** $(m/2, m^2/4)$ **15.** $a \leqslant e^{1/e}$

19. (a) $T_1 = D/c_1$, $T_2 = (2h\sec\theta)/c_1 + (D - 2h\tan\theta)/c_2$, $T_3 = \sqrt{4h^2 + D^2}/c_1$

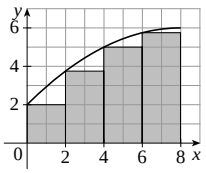
(c) $c_1 \approx 3.85$ km/s, $c_2 \approx 7.66$ km/s, $h \approx 0.42$ km

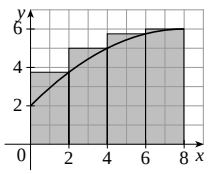
23. $3/\left(\sqrt[3]{2} - 1\right) \approx 11\frac{1}{2}$ h

CHAPTER 5

EXERCISES 5.1 ■ PAGE 369

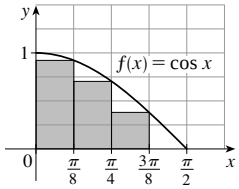
1. (a) $L_4 = 33$, $R_4 = 41$

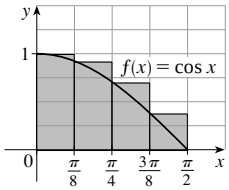




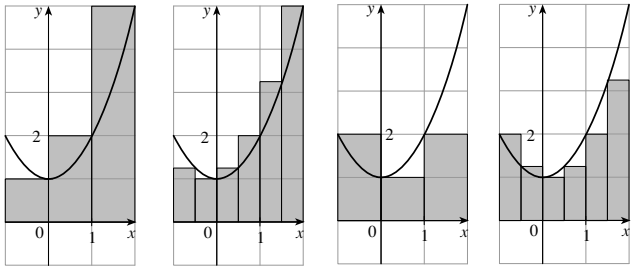
(b) $L_8 \approx 35.2$, $R_8 \approx 39.2$

3. (a) 0.7908, underestimate (b) 1.1835, overestimate

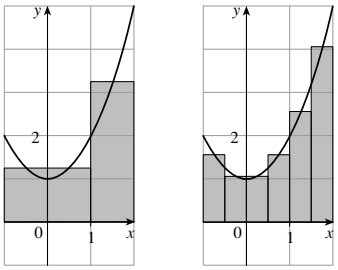




5. (a) 8, 6.875 (b) 5, 5.375

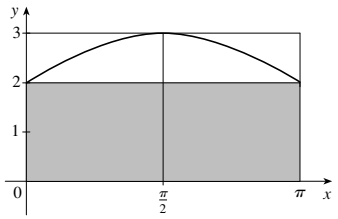


(c) 5.75, 5.9375

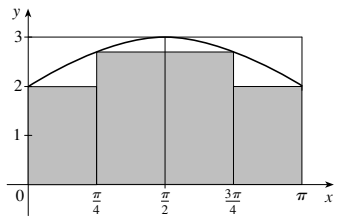


(d) M_6

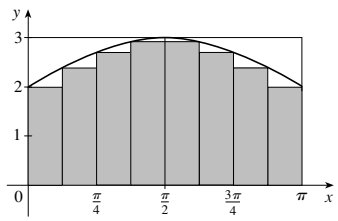
7. $n = 2$: upper $= 3\pi \approx 9.42$, lower $= 2\pi \approx 6.28$



$n = 4$: upper $= \left(10 + \sqrt{2}\right)(\pi/4) \approx 8.96$,
lower $= \left(8 + \sqrt{2}\right)(\pi/4) \approx 7.39$



$n = 8$: upper ≈ 8.65, lower ≈ 7.86



9. 0.2533, 0.2170, 0.2101, 0.2050; 0.2

11. (a) Left: 0.8100, 0.7937, 0.7904; right: 0.7600, 0.7770, 0.7804

13. 34.7 ft, 44.8 ft **15.** 63.2 L, 70 L **17.** 155 ft

19. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(1+2i/n)}{(1+2i/n)^2+1} \cdot \frac{2}{n}$ **21.** $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\sin(\pi i/n)} \cdot \frac{\pi}{n}$

23. The region under the graph of $y = \tan x$ from 0 to $\pi/4$

25. (a) $L_n < A < R_n$

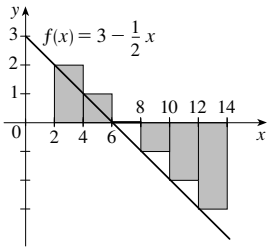
27. (a) $\lim_{n\to\infty} \frac{64}{n^6} \sum_{i=1}^{n} i^5$ (b) $\frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ (c) $\frac{32}{3}$

29. $\sin b$, 1

EXERCISES 5.2 ■ PAGE 382

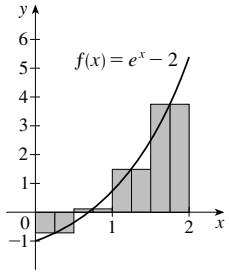
1. -6

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis; that is, the net area of the rectangles with respect to the x-axis.



3. 2.322986

The Riemann sum represents the sum of the areas of the three rectangles above the x-axis minus the area of the rectangle below the x-axis.



5. (a) 6 (b) 4 (c) 2

7. Lower, $L_5 = -64$; upper, $R_5 = 16$

9. 6.1820 **11.** 0.9071 **13.** 0.9029, 0.9018

15.

n	R_n
5	1.933766
10	1.983524
50	1.999342
100	1.999836

The values of R_n appear to be approaching 2.

17. $\int_2^6 x \ln(1+x^2)\,dx$ **19.** $\int_2^7 (5x^3 - 4x)\,dx$

21. -9 **23.** $\frac{2}{3}$ **25.** $-\frac{3}{4}$

29. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2+4i/n}{1+(2+4i/n)^5} \cdot \frac{4}{n}$

31. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sin \frac{5\pi i}{n}\right) \frac{\pi}{n} = \frac{2}{5}$

33. (a) 4 (b) 10 (c) -3 (d) 2

35. $\frac{3}{2}$ **37.** $3 + \frac{9}{4}\pi$ **39.** $\frac{5}{2}$ **41.** 0 **43.** 3

45. $e^5 - e^3$ **47.** $\int_{-1}^{5} f(x)\,dx$ **49.** 122

51. B < E < A < D < C **53.** 15

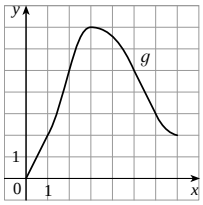
59. $3 \le \int_1^4 \sqrt{x}\,dx \le 6$ **61.** $\frac{\pi}{12} \le \int_{\pi/4}^{\pi/3} \tan x\,dx \le \frac{\pi}{12}\sqrt{3}$

63. $0 \le \int_0^2 xe^{-x}\,dx \le 2/e$ **71.** $\int_0^1 x^4\,dx$ **73.** $\frac{1}{2}$

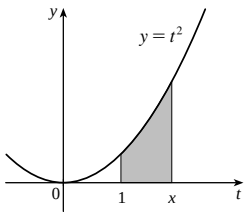
EXERCISES 5.3 ■ PAGE 394

1. One process undoes what the other one does. See the Fundamental Theorem of Calculus, page 393.

3. (a) 0, 2, 5, 7, 3
(b) (0, 3)
(c) $x = 3$
(d)



5.



(a), (b) x^2

7. $g'(x) = 1/(x^3+1)$ **9.** $g'(s) = (s - s^2)^8$

11. $F'(x) = -\sqrt{1+\sec x}$ **13.** $h'(x) = xe^x$

15. $y' = \sqrt{\tan x + \sqrt{\tan x}}\,\sec^2 x$

17. $y' = \frac{3(1-3x)^3}{1+(1-3x)^2}$ **19.** $\frac{3}{4}$ **21.** 63 **23.** $\frac{52}{3}$

25. $1 + \sqrt{3}/2$ **27.** $-\frac{37}{6}$ **29.** $\frac{40}{3}$ **31.** 1 **33.** $\frac{49}{3}$

35. $\ln 2 + 7$ **37.** $\frac{1}{e+1} + e - 1$ **39.** $4\pi/3$

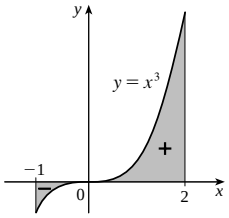
41. $e^2 - 1$ **43.** 0

45. The function $f(x) = x^{-4}$ is not continuous on the interval $[-2, 1]$, so FTC2 cannot be applied.

47. The function $f(\theta) = \sec\theta \tan\theta$ is not continuous on the interval $[\pi/3, \pi]$, so FTC2 cannot be applied.

49. $\frac{243}{4}$ **51.** 2

53. 3.75



55. $g'(x) = \frac{-2(4x^2-1)}{4x^2+1} + \frac{3(9x^2-1)}{9x^2+1}$

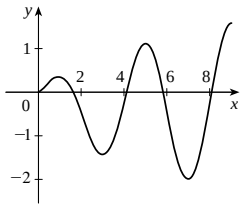
57. $F'(x) = 2xe^{x^4} - e^{x^2}$

59. $y' = \sin x \ln(1+2\cos x) + \cos x \ln(1+2\sin x)$

61. $(-4, 0)$ **63.** 29

65. (a) $-2\sqrt{n}$, $\sqrt{4n-2}$, n an integer > 0
(b) $(0, 1)$, $\left(-\sqrt{4n-1}, -\sqrt{4n-3}\right)$, and $\left(\sqrt{4n-1}, \sqrt{4n+1}\right)$, n an integer > 0 (c) 0.74

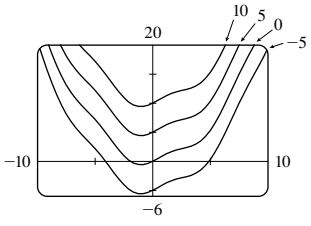
67. (a) Loc max at 1 and 5; loc min at 3 and 7
(b) $x = 9$
(c) $\left(\frac{1}{2}, 2\right)$, (4, 6), (8, 9)
(d) See graph at right.



69. $\frac{1}{4}$ **77.** $f(x) = x^{3/2}$, $a = 9$
79. (b) Average expenditure over $[0, t]$; minimize average expenditure

EXERCISES 5.4 ■ PAGE 403

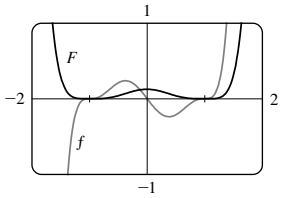
5. $\frac{1}{3}x^3 - (1/x) + C$ **7.** $\frac{1}{5}x^5 - \frac{1}{8}x^4 + \frac{1}{8}x^2 - 2x + C$
9. $\frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + C$ **11.** $\frac{1}{3}x^3 - 4\sqrt{x} + C$
13. $-\cos x + \cosh x + C$ **15.** $\frac{1}{2}\theta^2 + \csc\theta + C$
17. $\tan\alpha + C$
19. $\sin x + \frac{1}{4}x^2 + C$

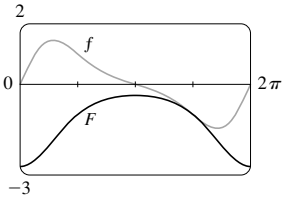


21. $-\frac{10}{3}$ **23.** $\frac{21}{5}$ **25.** -2 **27.** $5e^{\pi} + 1$ **29.** 36
31. $\frac{55}{63}$ **33.** $\frac{3}{4} - 2\ln 2$ **35.** $\dfrac{1}{11} + \dfrac{9}{\ln 10}$ **37.** $1 + \pi/4$
39. $\frac{256}{5}$ **41.** $\pi/3$ **43.** $\pi/6$ **45.** -3.5
47. ≈ 1.36 **49.** $\frac{4}{3}$
51. The increase in the child's weight (in pounds) between the ages of 5 and 10
53. Number of gallons of oil leaked in the first 2 hours
55. Increase in revenue when production is increased from 1000 to 5000 units
57. Newton-meters **59.** (a) $-\frac{3}{2}$ m (b) $\frac{41}{6}$ m
61. (a) $v(t) = \frac{1}{2}t^2 + 4t + 5$ m/s (b) $416\frac{2}{3}$ m
63. $46\frac{2}{3}$ kg **65.** 1.4 mi **67.** \$58,000
69. 5443 bacteria **71.** 4.75×10^5 megawatt-hours

EXERCISES 5.5 ■ PAGE 413

1. $-e^{-x} + C$ **3.** $\frac{2}{9}(x^3 + 1)^{3/2} + C$ **5.** $-\frac{1}{4}\cos^4\theta + C$
7. $-\frac{1}{2}\cos(x^2) + C$ **9.** $-\frac{1}{20}(1 - 2x)^{10} + C$
11. $\frac{1}{3}(2x + x^2)^{3/2} + C$ **13.** $-\frac{1}{3}\ln|5 - 3x| + C$
15. $-(1/\pi)\cos\pi t + C$ **17.** $\dfrac{1}{1 - e^u} + C$
19. $\frac{2}{3}\sqrt{3ax + bx^3} + C$ **21.** $\frac{1}{3}(\ln x)^3 + C$ **23.** $\frac{1}{4}\tan^4\theta + C$
25. $\frac{2}{3}(1 + e^x)^{3/2} + C$ **27.** $\frac{1}{15}(x^3 + 3x)^5 + C$
29. $-\dfrac{1}{\ln 5}\cos(5^t) + C$ **31.** $e^{\tan x} + C$ **33.** $-\dfrac{1}{\sin x} + C$
35. $-\frac{2}{3}(\cot x)^{3/2} + C$ **37.** $\frac{1}{3}\sinh^3 x + C$
39. $-\ln(1 + \cos^2 x) + C$ **41.** $\ln|\sin x| + C$
43. $\ln|\sin^{-1}x| + C$ **45.** $\tan^{-1}x + \frac{1}{2}\ln(1 + x^2) + C$
47. $\frac{1}{40}(2x + 5)^{10} - \frac{5}{36}(2x + 5)^9 + C$
49. $\frac{1}{8}(x^2 - 1)^4 + C$ **51.** $-e^{\cos x} + C$





53. $2/\pi$ **55.** $\frac{45}{28}$ **57.** 4 **59.** $e - \sqrt{e}$ **61.** 0
63. 3 **65.** $\frac{1}{3}\left(2\sqrt{2} - 1\right)a^3$ **67.** $\frac{16}{15}$ **69.** 2
71. $\ln(e + 1)$ **73.** $\frac{1}{6}$ **75.** $\sqrt{3} - \frac{1}{3}$ **77.** 6π
79. All three areas are equal. **81.** ≈ 4512 L
83. $\dfrac{5}{4\pi}\left(1 - \cos\dfrac{2\pi t}{5}\right)$ L **85.** 5 **91.** $\pi^2/4$

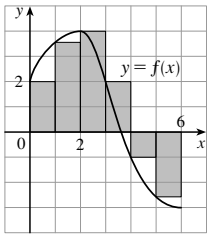
CHAPTER 5 REVIEW ■ PAGE 416

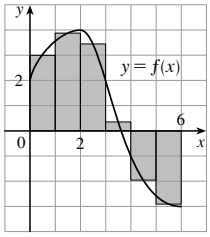
True-False Quiz

1. True **3.** True **5.** False **7.** True **9.** True
11. False **13.** True **15.** False **17.** False

Exercises

1. (a) 8 (b) 5.7





3. $\frac{1}{2} + \pi/4$ **5.** 3 **7.** f is c, f' is b, $\int_0^x f(t)\,dt$ is a
9. 37 **11.** $\frac{9}{10}$ **13.** -76 **15.** $\frac{21}{4}$ **17.** Does not exist
19. $\frac{1}{3}\sin 1$ **21.** 0 **23.** $-(1/x) - 2\ln|x| + x + C$
25. $\sqrt{x^2 + 4x} + C$ **27.** $\dfrac{1}{2\pi}\sin^2\pi t + C$
29. $2e^{\sqrt{x}} + C$ **31.** $-\frac{1}{2}[\ln(\cos x)]^2 + C$
33. $\frac{1}{4}\ln(1 + x^4) + C$ **35.** $\ln|1 + \sec\theta| + C$ **37.** $\frac{23}{3}$
39. $2\sqrt{1 + \sin x} + C$ **41.** $\frac{64}{5}$ **43.** $F'(x) = x^2/(1 + x^3)$
45. $g'(x) = 4x^3\cos(x^8)$ **47.** $y' = \left(2e^x - e^{\sqrt{x}}\right)/(2x)$
49. $4 \leq \int_1^3 \sqrt{x^2 + 3}\,dx \leq 4\sqrt{3}$ **55.** 0.280981
57. Number of barrels of oil consumed from Jan. 1, 2000, through Jan. 1, 2008

59. 72,400 **61.** 3 **63.** $c \approx 1.62$
65. $e^{2x}(2x-1)/(1-e^{-x})$ **71.** $\frac{2}{3}$

PROBLEMS PLUS ■ PAGE 420

1. $\pi/2$ **3.** $2k$ **5.** -1 **7.** e^{-2} **9.** $[-1, 2]$
11. (a) $\frac{1}{2}(n-1)n$
(b) $\frac{1}{2}[\![b]\!](2b - [\![b]\!] - 1) - \frac{1}{2}[\![a]\!](2a - [\![a]\!] - 1)$
17. $2(\sqrt{2}-1)$

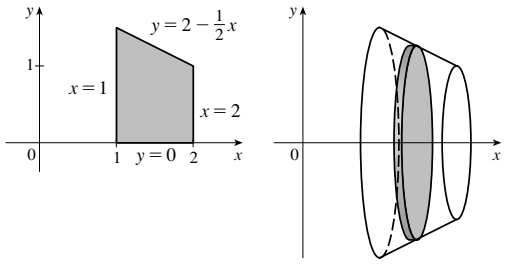
CHAPTER 6

EXERCISES 6.1 ■ PAGE 427

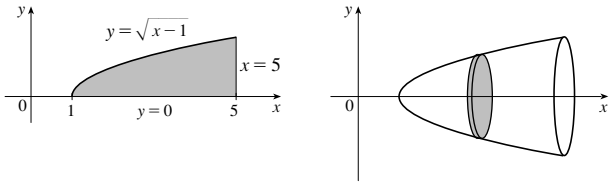
1. $\frac{32}{3}$ **3.** $e - (1/e) + \frac{10}{3}$ **5.** $e - (1/e) + \frac{4}{3}$ **7.** $\frac{9}{2}$
9. $\ln 2 - \frac{1}{2}$ **11.** $\frac{8}{3}$ **13.** 72 **15.** $e - 2$ **17.** $\frac{32}{3}$
19. $2/\pi + \frac{2}{3}$ **21.** $2 - 2\ln 2$ **23.** $\frac{1}{2}$ **25.** $\frac{59}{12}$ **27.** $\ln 2$
29. $\frac{5}{2}$ **31.** $\frac{3}{2}\sqrt{3} - 1$ **33.** 0, 0.90; 0.04
35. -1.11, 1.25, 2.86; 8.38 **37.** 2.80123 **39.** 0.25142
41. $12\sqrt{6} - 9$ **43.** $117\frac{1}{3}$ ft **45.** 4232 cm^2
47. (a) Car A (b) The distance by which A is ahead of B after 1 minute (c) Car A (d) $t \approx 2.2$ min
49. $\frac{24}{5}\sqrt{3}$ **51.** $4^{2/3}$ **53.** ± 6
55. $0 < m < 1$; $m - \ln m - 1$

EXERCISES 6.2 ■ PAGE 438

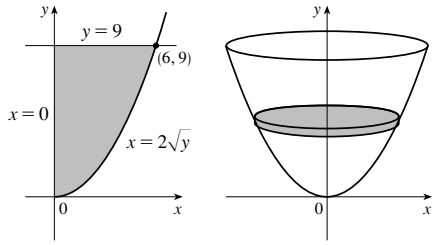
1. $19\pi/12$



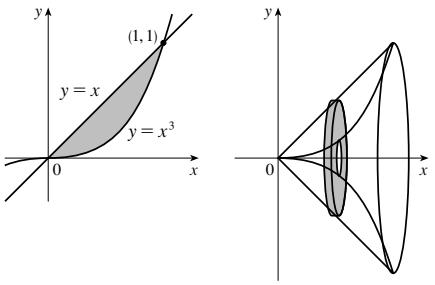
3. 8π



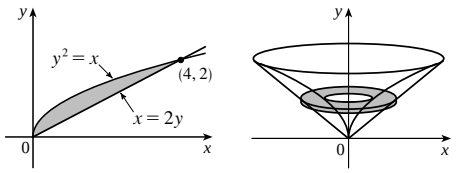
5. 162π



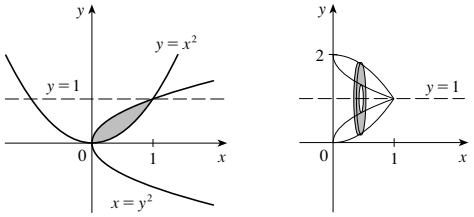
7. $4\pi/21$



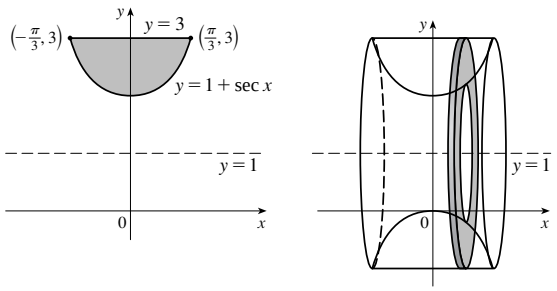
9. $64\pi/15$



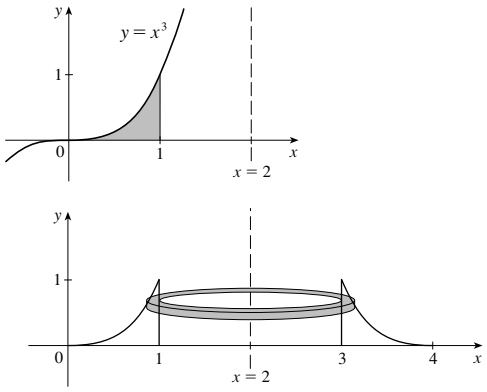
11. $11\pi/30$



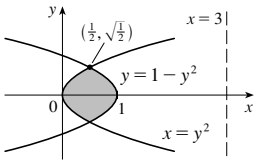
13. $2\pi(\frac{4}{3}\pi - \sqrt{3})$

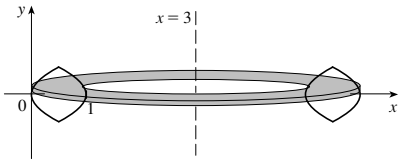


15. $3\pi/5$



17. $10\sqrt{2}\,\pi/3$

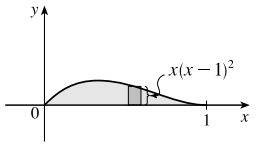


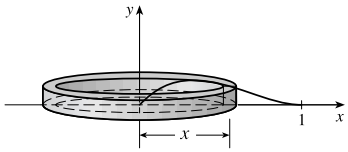


19. $\pi/3$ **21.** $\pi/3$ **23.** $\pi/3$
25. $13\pi/45$ **27.** $\pi/3$ **29.** $17\pi/45$
31. (a) $\int_{-1}^{1} \pi\left(e^{-x^2}\right)^2\,dx \approx 3.75825$
(b) $\int_{-1}^{1} \pi\{[e^{-x^2} - (-1)]^2 - [0 - (-1)]^2\}\,dx \approx 13.14312$
33. (a)
$\int_{-2}^{2} \pi\left\{\left[2 - \left(-\sqrt{1 - x^2/4}\right)\right]^2 - \left(2 - \sqrt{1 - x^2/4}\right)^2\right\}\,dx \approx 78.95684$
(b) $\int_{-1}^{1} \pi\left\{\left[2 - \left(-\sqrt{4 - 4y^2}\right)\right]^2 - \left(2 - \sqrt{4 - 4y^2}\right)^2\right\}\,dx \approx 78.95684$
35. $-1.288,\ 0.884;\ 23.780$ **37.** $\frac{11}{8}\pi^2$
39. Solid obtained by rotating the region $0 \leq x \leq \pi$, $0 \leq y \leq \sqrt{\sin x}$ about the x-axis
41. Solid obtained by rotating the region above the x-axis bounded by $x = y^2$ and $x = y^4$ about the y-axis
43. 1110 cm^3 **45.** (a) 196 (b) 838
47. $\frac{1}{3}\pi r^2 h$ **49.** $\pi h^2\left(r - \frac{1}{3}h\right)$ **51.** $\frac{2}{3}b^2 h$
53. 10 cm^3 **55.** 24 **57.** $\frac{1}{3}$ **59.** $\frac{8}{15}$
61. (a) $8\pi R \int_0^r \sqrt{r^2 - y^2}\,dy$ (b) $2\pi^2 r^2 R$
63. (b) $\pi r^2 h$ **65.** $\frac{5}{12}\pi r^3$ **67.** $8\int_0^r \sqrt{R^2 - y^2}\sqrt{r^2 - y^2}\,dy$

EXERCISES 6.3 ■ PAGE 444

1. Circumference $= 2\pi x$, height $= x(x-1)^2$; $\pi/15$





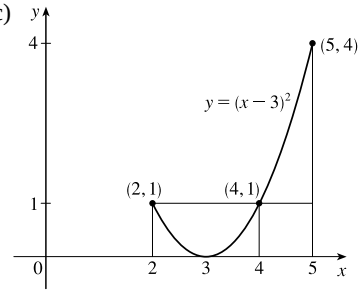
3. $6\pi/7$ **5.** $\pi(1 - 1/e)$ **7.** 8π **9.** 4π **11.** $768\pi/7$
13. $16\pi/3$ **15.** $7\pi/15$ **17.** $8\pi/3$ **19.** $5\pi/14$
21. (a) $\int_0^2 2\pi x(xe^{-x})\,dx$ (b) 4.06300
23. (a) $\int_{-\pi/2}^{\pi/2} 2\pi(\pi - x)[\cos^4 x - (-\cos^4 x)]\,dx$ (b) 46.50942
25. (a) $\int_0^{\pi} 2\pi(4 - y)\sqrt{\sin y}\,dy$ (b) 36.57476
27. 3.68
29. Solid obtained by rotating the region $0 \leq y \leq x^4$, $0 \leq x \leq 3$ about the y-axis
31. Solid obtained by rotating the region bounded by (i) $x = 1 - y^2$, $x = 0$, and $y = 0$, or (ii) $x = y^2$, $x = 1$, and $y = 0$ about the line $y = 3$
33. 0.13 **35.** $\frac{1}{32}\pi^3$ **37.** 8π **39.** $4\sqrt{3}\,\pi$ **41.** $4\pi/3$
43. $117\pi/5$ **45.** $\frac{4}{3}\pi r^3$ **47.** $\frac{1}{3}\pi r^2 h$

EXERCISES 6.4 ■ PAGE 449

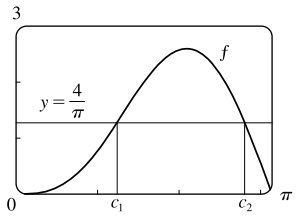
1. (a) 7200 ft-lb (b) 7200 ft-lb
3. 4.5 ft-lb **5.** 180 J **7.** $\frac{15}{4}$ ft-lb
9. (a) $\frac{25}{24} \approx 1.04$ J (b) 10.8 cm **11.** $W_2 = 3W_1$
13. (a) 625 ft-lb (b) $\frac{1875}{4}$ ft-lb **15.** 650,000 ft-lb
17. 3857 J **19.** 2450 J **21.** $\approx 1.06 \times 10^6$ J
23. $\approx 1.04 \times 10^5$ ft-lb **25.** 2.0 m
29. (a) $Gm_1m_2\left(\frac{1}{a} - \frac{1}{b}\right)$ (b) $\approx 8.50 \times 10^9$ J

EXERCISES 6.5 ■ PAGE 453

1. $\frac{8}{3}$ **3.** $\frac{45}{28}$ **5.** $(2/\pi)(e - 1)$ **7.** $2/(5\pi)$
9. (a) 1 (b) 2, 4 (c)



11. (a) $4/\pi$ (b) $\approx 1.24, 2.81$
(c)



15. $\frac{9}{8}$ **17.** $(50 + 28/\pi)°\text{F} \approx 59°\text{F}$ **19.** 6 kg/m
21. About 4056 million (or 4 billion) people
23. $5/(4\pi) \approx 0.4$ L

CHAPTER 6 REVIEW ■ PAGE 457

Exercises

1. $\frac{8}{3}$ **3.** $\frac{7}{12}$ **5.** $\frac{4}{3} + 4/\pi$ **7.** $64\pi/15$ **9.** $1656\pi/5$
11. $\frac{4}{3}\pi(2ah + h^2)^{3/2}$ **13.** $\int_{-\pi/3}^{\pi/3} 2\pi\left(\pi/2 - x\right)\left(\cos^2 x - \frac{1}{4}\right)\,dx$
15. (a) $2\pi/15$ (b) $\pi/6$ (c) $8\pi/15$
17. (a) 0.38 (b) 0.87
19. Solid obtained by rotating the region $0 \leq y \leq \cos x$, $0 \leq x \leq \pi/2$ about the y-axis
21. Solid obtained by rotating the region $0 \leq x \leq \pi$, $0 \leq y \leq 2 - \sin x$ about the x-axis
23. 36 **25.** $\frac{125}{3}\sqrt{3}\ \text{m}^3$ **27.** 3.2 J
29. (a) $8000\pi/3 \approx 8378$ ft-lb (b) 2.1 ft
31. $f(x)$

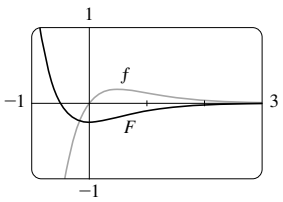
PROBLEMS PLUS ■ PAGE 459

1. (a) $f(t) = 3t^2$ (b) $f(x) = \sqrt{2x/\pi}$ **3.** $\frac{32}{27}$
5. (b) 0.2261 (c) 0.6736 m
(d) (i) $1/(105\pi) \approx 0.003$ in/s (ii) $370\pi/3$ s ≈ 6.5 min
9. $y = \frac{32}{9}x^2$
11. (a) $V = \int_0^h \pi[f(y)]^2\,dy$
(c) $f(y) = \sqrt{kA/(\pi C)}\,y^{1/4}$. Advantage: the markings on the container are equally spaced.
13. $b = 2a$ **15.** $B = 16A$

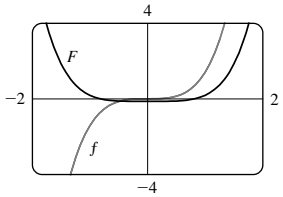
CHAPTER 7

EXERCISES 7.1 ■ PAGE 468

1. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ **3.** $\frac{1}{5}x \sin 5x + \frac{1}{25}\cos 5x + C$
5. $-\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$
7. $(x^2 + 2x)\sin x + (2x + 2)\cos x - 2\sin x + C$
9. $x \ln \sqrt[3]{x} - \frac{1}{3}x + C$ **11.** $t \arctan 4t - \frac{1}{8}\ln(1 + 16t^2) + C$
13. $\frac{1}{2}t \tan 2t - \frac{1}{4}\ln|\sec 2t| + C$
15. $x(\ln x)^2 - 2x \ln x + 2x + C$
17. $\frac{1}{13}e^{2\theta}(2\sin 3\theta - 3\cos 3\theta) + C$
19. $z^3e^z - 3z^2e^z + 6ze^z - 6e^z + C$
21. $\dfrac{e^{2x}}{4(2x + 1)} + C$ **23.** $\dfrac{\pi - 2}{2\pi^2}$
25. $1 - 1/e$ **27.** $\frac{81}{4}\ln 3 - 5$ **29.** $\frac{1}{4} - \frac{3}{4}e^{-2}$
31. $\frac{1}{6}\left(\pi + 6 - 3\sqrt{3}\right)$ **33.** $\sin x\,(\ln \sin x - 1) + C$
35. $\frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{62}{125}$
37. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$ **39.** $-\frac{1}{2} - \pi/4$
41. $\frac{1}{2}(x^2 - 1)\ln(1 + x) - \frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4} + C$
43. $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$



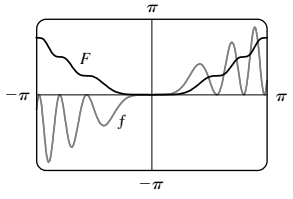
45. $\frac{1}{3}x^2(1 + x^2)^{3/2} - \frac{2}{15}(1 + x^2)^{5/2} + C$



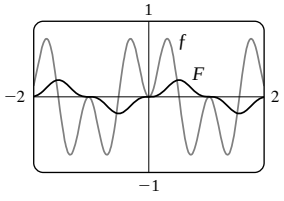
47. (b) $-\frac{1}{4}\cos x \sin^3 x + \frac{3}{8}x - \frac{3}{16}\sin 2x + C$
49. (b) $\frac{2}{3}, \frac{8}{15}$
55. $x[(\ln x)^3 - 3(\ln x)^2 + 6\ln x - 6] + C$
57. $\frac{16}{3}\ln 2 - \frac{29}{9}$ **59.** $-1.75119,\ 1.17210;\ 3.99926$
61. $4 - 8/\pi$ **63.** $2\pi e$ **65.** $1 - (2/\pi)\ln 2$
67. $2 - e^{-t}(t^2 + 2t + 2)$ m **69.** 2

EXERCISES 7.2 ■ PAGE 476

1. $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$ **3.** $\frac{1}{120}$
5. $\dfrac{1}{3\pi}\sin^3(\pi x) - \dfrac{2}{5\pi}\sin^5(\pi x) + \dfrac{1}{7\pi}\sin^7(\pi x) + C$
7. $\pi/4$ **9.** $3\pi/8$ **11.** $\pi/16$
13. $\frac{1}{4}t^2 - \frac{1}{4}t\sin 2t - \frac{1}{8}\cos 2t + C$
15. $\frac{2}{45}\sqrt{\sin\alpha}\,(45 - 18\sin^2\alpha + 5\sin^4\alpha) + C$
17. $\frac{1}{2}\cos^2 x - \ln|\cos x| + C$ **19.** $\ln|\sin x| + 2\sin x + C$
21. $\frac{1}{3}\sec^3 x + C$ **23.** $\tan x - x + C$
25. $\frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$ **27.** $\frac{117}{8}$
29. $\frac{1}{3}\sec^3 x - \sec x + C$
31. $\frac{1}{4}\sec^4 x - \tan^2 x + \ln|\sec x| + C$
33. $x\sec x - \ln|\sec x + \tan x| + C$ **35.** $\sqrt{3} - \frac{1}{3}\pi$
37. $\frac{22}{105}\sqrt{2} - \frac{8}{105}$ **39.** $\ln|\csc x - \cot x| + C$
41. $-\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C$ **43.** $\frac{1}{8}\sin 4\theta - \frac{1}{12}\sin 6\theta + C$
45. $\frac{1}{2}\sqrt{2}$ **47.** $\frac{1}{2}\sin 2x + C$
49. $x\tan x - \ln|\sec x| - \frac{1}{2}x^2 + C$
51. $\frac{1}{4}x^2 - \frac{1}{4}\sin(x^2)\cos(x^2) + C$



53. $\frac{1}{6}\sin 3x - \frac{1}{18}\sin 9x + C$



55. 0 **57.** 1 **59.** 0 **61.** $\pi^2/4$ **63.** $\pi\left(2\sqrt{2} - \frac{5}{2}\right)$
65. $s = (1 - \cos^3\omega t)/(3\omega)$

EXERCISES 7.3 ■ PAGE 483

1. $-\dfrac{\sqrt{4 - x^2}}{4x} + C$ **3.** $\sqrt{x^2 - 4} - 2\sec^{-1}\left(\dfrac{x}{2}\right) + C$
5. $\dfrac{\pi}{24} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$ **7.** $\dfrac{1}{\sqrt{2}\,a^2}$
9. $\ln\left(\sqrt{x^2 + 16} + x\right) + C$ **11.** $\frac{1}{4}\sin^{-1}(2x) + \frac{1}{2}x\sqrt{1 - 4x^2} + C$
13. $\frac{1}{6}\sec^{-1}(x/3) - \sqrt{x^2 - 9}/(2x^2) + C$
15. $\frac{1}{16}\pi a^4$ **17.** $\sqrt{x^2 - 7} + C$
19. $\ln\left|\left(\sqrt{1 + x^2} - 1\right)/x\right| + \sqrt{1 + x^2} + C$ **21.** $\frac{9}{500}\pi$
23. $\frac{9}{2}\sin^{-1}((x - 2)/3) + \frac{1}{2}(x - 2)\sqrt{5 + 4x - x^2} + C$
25. $\sqrt{x^2 + x + 1} - \frac{1}{2}\ln\left(\sqrt{x^2 + x + 1} + x + \frac{1}{2}\right) + C$
27. $\frac{1}{2}(x + 1)\sqrt{x^2 + 2x} - \frac{1}{2}\ln\left|x + 1 + \sqrt{x^2 + 2x}\right| + C$
29. $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1 - x^4} + C$
33. $\frac{1}{6}\left(\sqrt{48} - \sec^{-1} 7\right)$ **37.** $\frac{3}{8}\pi^2 + \frac{3}{4}\pi$
41. $2\pi^2 Rr^2$ **43.** $r\sqrt{R^2 - r^2} + \pi r^2/2 - R^2 \arcsin(r/R)$

EXERCISES 7.4 ■ PAGE 492

1. (a) $\dfrac{A}{4x-3}+\dfrac{B}{2x+5}$ (b) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{5-2x}$

3. (a) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{Dx+E}{x^2+4}$

(b) $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x-3}+\dfrac{D}{(x-3)^2}$

5. (a) $x^4+4x^2+16+\dfrac{A}{x+2}+\dfrac{B}{x-2}$

(b) $\dfrac{Ax+B}{x^2-x+1}+\dfrac{Cx+D}{x^2+2}+\dfrac{Ex+F}{(x^2+2)^2}$

7. $\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+\ln|x-1|+C$

9. $\frac{1}{2}\ln|2x+1|+2\ln|x-1|+C$ **11.** $2\ln\frac{3}{2}$

13. $a\ln|x-b|+C$ **15.** $\frac{7}{6}+\ln\frac{2}{3}$

17. $\frac{27}{5}\ln 2-\frac{9}{5}\ln 3$ $\left(\text{or } \frac{9}{5}\ln\frac{8}{3}\right)$

19. $10\ln|x-3|-9\ln|x-2|+\dfrac{5}{x-2}+C$

21. $\frac{1}{2}x^2-2\ln(x^2+4)+2\tan^{-1}(x/2)+C$

23. $\ln|x-1|-\frac{1}{2}\ln(x^2+9)-\frac{1}{3}\tan^{-1}(x/3)+C$

25. $-2\ln|x+1|+\ln(x^2+1)+2\tan^{-1}x+C$

27. $\frac{1}{2}\ln(x^2+1)+\left(1/\sqrt{2}\right)\tan^{-1}\left(x/\sqrt{2}\right)+C$

29. $\frac{1}{2}\ln(x^2+2x+5)+\frac{3}{2}\tan^{-1}\left(\dfrac{x+1}{2}\right)+C$

31. $\frac{1}{3}\ln|x-1|-\frac{1}{6}\ln(x^2+x+1)-\dfrac{1}{\sqrt{3}}\tan^{-1}\dfrac{2x+1}{\sqrt{3}}+C$

33. $\frac{1}{4}\ln\frac{8}{3}$ **35.** $\frac{1}{16}\ln|x|-\frac{1}{32}\ln(x^2+4)+\dfrac{1}{8(x^2+4)}+C$

37. $\frac{7}{8}\sqrt{2}\tan^{-1}\left(\dfrac{x-2}{\sqrt{2}}\right)+\dfrac{3x-8}{4(x^2-4x+6)}+C$

39. $2\sqrt{x+1}-\ln\left(\sqrt{x+1}+1\right)+\ln\left|\sqrt{x+1}-1\right|+C$

41. $-2\ln\sqrt{x}-\dfrac{2}{\sqrt{x}}+2\ln\left(\sqrt{x}+1\right)+C$

43. $\frac{3}{10}(x^2+1)^{5/3}-\frac{3}{4}(x^2+1)^{2/3}+C$

45. $2\sqrt{x}+3\sqrt[3]{x}+6\sqrt[6]{x}+6\ln\left|\sqrt[6]{x}-1\right|+C$

47. $\ln\dfrac{(e^x+2)^2}{e^x+1}+C$

49. $\ln|\tan t+1|-\ln|\tan t+2|+C$

51. $x-\ln(e^x+1)+C$

53. $\left(x-\frac{1}{2}\right)\ln(x^2-x+2)-2x+\sqrt{7}\tan^{-1}\left(\dfrac{2x-1}{\sqrt{7}}\right)+C$

55. $-\frac{1}{2}\ln 3\approx -0.55$

57. $\frac{1}{2}\ln\left|\dfrac{x-2}{x}\right|+C$ **61.** $\frac{1}{5}\ln\left|\dfrac{2\tan(x/2)-1}{\tan(x/2)+2}\right|+C$

63. $4\ln\frac{2}{3}+2$ **65.** $-1+\frac{11}{3}\ln 2$

67. $t=-\ln P-\frac{1}{9}\ln(0.9P+900)+C$, where $C\approx 10.23$

69. (a) $\dfrac{24{,}110}{4879}\dfrac{1}{5x+2}-\dfrac{668}{323}\dfrac{1}{2x+1}-\dfrac{9438}{80{,}155}\dfrac{1}{3x-7}+\dfrac{1}{260{,}015}\dfrac{22{,}098x+48{,}935}{x^2+x+5}$

(b) $\dfrac{4822}{4879}\ln|5x+2|-\dfrac{334}{323}\ln|2x+1|-\dfrac{3146}{80{,}155}\ln|3x-7|+\dfrac{11{,}049}{260{,}015}\ln(x^2+x+5)+\dfrac{75{,}772}{260{,}015\sqrt{19}}\tan^{-1}\dfrac{2x+1}{\sqrt{19}}+C$

The CAS omits the absolute value signs and the constant of integration.

73. $\dfrac{1}{a^n(x-a)}-\dfrac{1}{a^n x}-\dfrac{1}{a^{n-1}x^2}-\cdots-\dfrac{1}{ax^n}$

EXERCISES 7.5 ■ PAGE 499

1. $\sin x+\frac{1}{3}\sin^3 x+C$

3. $\sin x+\ln|\csc x-\cot x|+C$

5. $\dfrac{1}{2\sqrt{2}}\tan^{-1}\left(\dfrac{t^2}{\sqrt{2}}\right)+C$ **7.** $e^{\pi/4}-e^{-\pi/4}$

9. $\frac{243}{5}\ln 3-\frac{242}{25}$ **11.** $\frac{1}{2}\ln(x^2-4x+5)+\tan^{-1}(x-2)+C$

13. $-\frac{1}{5}\cos^5 t+\frac{2}{7}\cos^7 t-\frac{1}{9}\cos^9 t+C$ **15.** $x/\sqrt{1-x^2}+C$

17. $\frac{1}{4}\pi^2$ **19.** $e^{e^x}+C$ **21.** $(x+1)\arctan\sqrt{x}-\sqrt{x}+C$

23. $\frac{4097}{45}$ **25.** $3x+\frac{23}{3}\ln|x-4|-\frac{5}{3}\ln|x+2|+C$

27. $x-\ln(1+e^x)+C$

29. $x\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}+C$

31. $\sin^{-1}x-\sqrt{1-x^2}+C$

33. $2\sin^{-1}\left(\dfrac{x+1}{2}\right)+\dfrac{x+1}{2}\sqrt{3-2x-x^2}+C$

35. $\frac{1}{8}\sin 4x+\frac{1}{16}\sin 8x+C$ **37.** $\frac{1}{4}$

39. $\ln|\sec\theta-1|-\ln|\sec\theta|+C$

41. $\theta\tan\theta-\frac{1}{2}\theta^2-\ln|\sec\theta|+C$ **43.** $\frac{2}{3}\tan^{-1}(x^{3/2})+C$

45. $-\frac{1}{3}(x^3+1)e^{-x^3}+C$

47. $\ln|x-1|-3(x-1)^{-1}-\frac{3}{2}(x-1)^{-2}-\frac{1}{3}(x-1)^{-3}+C$

49. $\ln\left|\dfrac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1}\right|+C$ **51.** $-\ln\left|\dfrac{\sqrt{4x^2+1}+1}{2x}\right|+C$

53. $\dfrac{1}{m}x^2\cosh(mx)-\dfrac{2}{m^2}x\sinh(mx)+\dfrac{2}{m^3}\cosh(mx)+C$

55. $2\ln\sqrt{x}-2\ln\left(1+\sqrt{x}\right)+C$

57. $\frac{3}{7}(x+c)^{7/3}-\frac{3}{4}c(x+c)^{4/3}+C$

59. $\sin(\sin x)-\frac{1}{3}\sin^3(\sin x)+C$

61. $\csc\theta-\cot\theta+C$ or $\tan(\theta/2)+C$

63. $2\left(x-2\sqrt{x}+2\right)e^{\sqrt{x}}+C$

65. $-\tan^{-1}(\cos^2 x)+C$ **67.** $\frac{2}{3}[(x+1)^{3/2}-x^{3/2}]+C$

69. $\sqrt{2}-2/\sqrt{3}+\ln\left(2+\sqrt{3}\right)-\ln\left(1+\sqrt{2}\right)$

71. $e^x-\ln(1+e^x)+C$

73. $-\sqrt{1-x^2}+\frac{1}{2}(\arcsin x)^2+C$

75. $\frac{1}{8}\ln|x-2|-\frac{1}{16}\ln(x^2+4)-\frac{1}{8}\tan^{-1}(x/2)+C$

77. $2(x-2)\sqrt{1+e^x}+2\ln\dfrac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}+C$

79. $\frac{1}{3}x\sin^3 x+\frac{1}{3}\cos x-\frac{1}{9}\cos^3 x+C$

81. $2\sqrt{1+\sin x}+C$ **83.** $xe^{x^2}+C$

EXERCISES 7.6 ■ PAGE 504

1. $-\frac{5}{21}$ **3.** $\sqrt{13} - \frac{3}{4}\ln\left(4 + \sqrt{13}\right) - \frac{1}{2} + \frac{3}{4}\ln 3$

5. $\frac{\pi}{8} - \frac{1}{4}\ln\left(1 + \frac{1}{16}\pi^2\right)$ **7.** $\frac{1}{6}\ln\left|\frac{\sin x - 3}{\sin x + 3}\right| + C$

9. $-\sqrt{4x^2 + 9}/(9x) + C$ **11.** $e - 2$

13. $-\frac{1}{2}\tan^2(1/z) - \ln|\cos(1/z)| + C$

15. $\frac{1}{2}(e^{2x} + 1)\arctan(e^x) - \frac{1}{2}e^x + C$

17. $\frac{2y - 1}{8}\sqrt{6 + 4y - 4y^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y - 1}{\sqrt{7}}\right)$
$- \frac{1}{12}(6 + 4y - 4y^2)^{3/2} + C$

19. $\frac{1}{9}\sin^3 x\,[3\ln(\sin x) - 1] + C$ **21.** $\frac{1}{2\sqrt{3}}\ln\left|\frac{e^x + \sqrt{3}}{e^x - \sqrt{3}}\right| + C$

23. $\frac{1}{4}\tan x\sec^3 x + \frac{3}{8}\tan x\sec x + \frac{3}{8}\ln|\sec x + \tan x| + C$

25. $\frac{1}{2}(\ln x)\sqrt{4 + (\ln x)^2} + 2\ln\left[\ln x + \sqrt{4 + (\ln x)^2}\right] + C$

27. $-\frac{1}{2}x^{-2}\cos^{-1}(x^{-2}) + \frac{1}{2}\sqrt{1 - x^{-4}} + C$

29. $\sqrt{e^{2x} - 1} - \cos^{-1}(e^{-x}) + C$

31. $\frac{1}{5}\ln|x^5 + \sqrt{x^{10} - 2}| + C$ **33.** $\frac{3}{8}\pi^2$

37. $\frac{1}{3}\tan x\sec^2 x + \frac{2}{3}\tan x + C$

39. $\frac{1}{4}x(x^2 + 2)\sqrt{x^2 + 4} - 2\ln\left(\sqrt{x^2 + 4} + x\right) + C$

41. $\frac{1}{4}\cos^3 x\sin x + \frac{3}{8}x + \frac{3}{8}\sin x\cos x + C$

43. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \ln|\cos x| + C$

45. (a) $-\ln\left|\frac{1 + \sqrt{1 - x^2}}{x}\right| + C$;
both have domain $(-1, 0) \cup (0, 1)$

EXERCISES 7.7 ■ PAGE 516

1. (a) $L_2 = 6$, $R_2 = 12$, $M_2 \approx 9.6$
(b) L_2 is an underestimate, R_2 and M_2 are overestimates.
(c) $T_2 = 9 < I$ (d) $L_n < T_n < I < M_n < R_n$

3. (a) $T_4 \approx 0.895759$ (underestimate)
(b) $M_4 \approx 0.908907$ (overestimate)
$T_4 < I < M_4$

5. (a) $M_{10} \approx 0.806598$, $E_M \approx -0.001879$
(b) $S_{10} \approx 0.804779$, $E_S \approx -0.000060$

7. (a) 1.506361 (b) 1.518362 (c) 1.511519

9. (a) 2.660833 (b) 2.664377 (c) 2.663244

11. (a) 2.591334 (b) 2.681046 (c) 2.631976

13. (a) 4.513618 (b) 4.748256 (c) 4.675111

15. (a) −0.495333 (b) −0.543321 (c) −0.526123

17. (a) 8.363853 (b) 8.163298 (c) 8.235114

19. (a) $T_8 \approx 0.902333$, $M_8 \approx 0.905620$
(b) $|E_T| \leq 0.0078$, $|E_M| \leq 0.0039$
(c) $n = 71$ for T_n, $n = 50$ for M_n

21. (a) $T_{10} \approx 1.983524$, $E_T \approx 0.016476$;
$M_{10} \approx 2.008248$, $E_M \approx -0.008248$;
$S_{10} \approx 2.000110$, $E_S \approx -0.000110$
(b) $|E_T| \leq 0.025839$, $|E_M| \leq 0.012919$, $|E_S| \leq 0.000170$
(c) $n = 509$ for T_n, $n = 360$ for M_n, $n = 22$ for S_n

23. (a) 2.8 (b) 7.954926518 (c) 0.2894
(d) 7.954926521 (e) The actual error is much smaller.
(f) 10.9 (g) 7.953789422 (h) 0.0593
(i) The actual error is smaller. (j) $n \geq 50$

25.

n	L_n	R_n	T_n	M_n
5	0.742943	1.286599	1.014771	0.992621
10	0.867782	1.139610	1.003696	0.998152
20	0.932967	1.068881	1.000924	0.999538

n	E_L	E_R	E_T	E_M
5	0.257057	−0.286599	−0.014771	0.007379
10	0.132218	−0.139610	−0.003696	0.001848
20	0.067033	−0.068881	−0.000924	0.000462

Observations are the same as after Example 1.

27.

n	T_n	M_n	S_n
6	6.695473	6.252572	6.403292
12	6.474023	6.363008	6.400206

n	E_T	E_M	E_S
6	−0.295473	0.147428	−0.003292
12	−0.074023	0.036992	−0.000206

Observations are the same as after Example 1.

29. (a) 19.8 (b) 20.6 (c) $20.5\overline{3}$

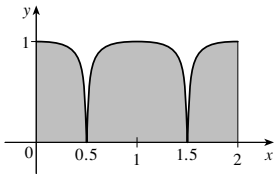
31. (a) 14.4 (b) $\frac{1}{2}$

33. 64.4°F **35.** $37.7\overline{3}$ ft/s **37.** 10,177 megawatt-hours

39. (a) 190 (b) 828

41. 6.0 **43.** 59.4

45.



EXERCISES 7.8 ■ PAGE 527

Abbreviations: C, convergent; D, divergent

1. (a), (d) Infinite discontinuity (b), (c) Infinite interval

3. $\frac{1}{2} - 1/(2t^2)$; 0.495, 0.49995, 0.4999995; 0.5

5. 2 **7.** D **9.** $\frac{1}{5}e^{-10}$ **11.** D **13.** 0 **15.** D

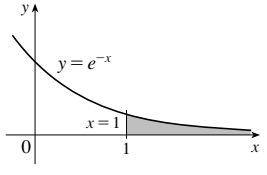
17. $\ln 2$ **19.** $-\frac{1}{4}$ **21.** D **23.** $\pi/9$ **25.** $\frac{1}{2}$ **27.** D

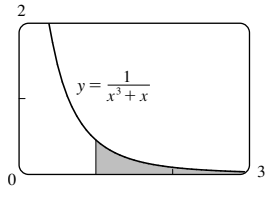
29. $\frac{32}{3}$ **31.** D **33.** $\frac{9}{2}$ **35.** D **37.** $-2/e$

39. $\frac{8}{3}\ln 2 - \frac{8}{9}$

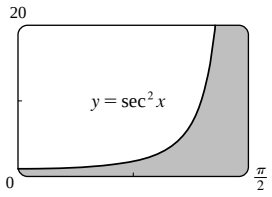
41. $1/e$

43. $\frac{1}{2}\ln 2$





45. Infinite area

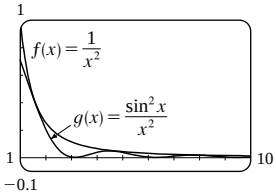


47. (a)

t	$\int_1^t [(\sin^2 x)/x^2]\,dx$
2	0.447453
5	0.577101
10	0.621306
100	0.668479
1,000	0.672957
10,000	0.673407

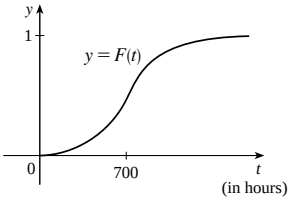
It appears that the integral is convergent.

(c)



49. C **51.** D **53.** D **55.** π **57.** $p < 1,\ 1/(1-p)$
59. $p > -1,\ -1/(p+1)^2$ **65.** $\sqrt{2GM/R}$

67. (a)



(b) The rate at which the fraction $F(t)$ increases as t increases
(c) 1; all bulbs burn out eventually

69. 1000

71. (a) $F(s) = 1/s,\ s > 0$ (b) $F(s) = 1/(s-1),\ s > 1$
(c) $F(s) = 1/s^2,\ s > 0$

77. $C = 1$; $\ln 2$ **79.** No

CHAPTER 7 REVIEW ■ PAGE 530

True-False Quiz

1. False **3.** False **5.** False **7.** False
9. (a) True (b) False **11.** False **13.** False

Exercises

1. $\frac{7}{2} + \ln 2$ **3.** $e - 1$ **5.** $\ln|2t+1| - \ln|t+1| + C$
7. $\frac{2}{15}$ **9.** $-\cos(\ln t) + C$ **11.** $\sqrt{3} - \frac{1}{3}\pi$
13. $3e^{\sqrt[3]{x}}(x^{2/3} - 2x^{1/3} + 2) + C$
15. $-\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| + C$
17. $x \sec x - \ln|\sec x + \tan x| + C$
19. $\frac{1}{18}\ln(9x^2+6x+5) + \frac{1}{9}\tan^{-1}\left[\frac{1}{2}(3x+1)\right] + C$
21. $\ln|x - 2 + \sqrt{x^2-4x}| + C$
23. $\ln\left|\dfrac{\sqrt{x^2+1}-1}{x}\right| + C$
25. $\frac{3}{2}\ln(x^2+1) - 3\tan^{-1}x + \sqrt{2}\tan^{-1}\left(x/\sqrt{2}\right) + C$
27. $\frac{2}{5}$ **29.** 0 **31.** $6 - \frac{3}{2}\pi$
33. $\dfrac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\dfrac{x}{2}\right) + C$
35. $4\sqrt{1+\sqrt{x}} + C$ **37.** $\frac{1}{2}\sin 2x - \frac{1}{8}\cos 4x + C$
39. $\frac{1}{8}e - \frac{1}{4}$ **41.** $\frac{1}{36}$ **43.** D
45. $4\ln 4 - 8$ **47.** $-\frac{4}{3}$ **49.** $\pi/4$
51. $(x+1)\ln(x^2+2x+2) + 2\arctan(x+1) - 2x + C$
53. 0
55. $\frac{1}{4}(2x-1)\sqrt{4x^2-4x-3} - \ln|2x-1+\sqrt{4x^2-4x-3}| + C$
57. $\frac{1}{2}\sin x\sqrt{4+\sin^2 x} + 2\ln\left(\sin x + \sqrt{4+\sin^2 x}\right) + C$
61. No
63. (a) 1.925444 (b) 1.920915 (c) 1.922470
65. (a) 0.01348, $n \geqslant 368$ (b) 0.00674, $n \geqslant 260$
67. 8.6 mi
69. (a) 3.8 (b) 1.7867, 0.000646 (c) $n \geqslant 30$
71. (a) D (b) C
73. 2 **75.** $\frac{3}{16}\pi^2$

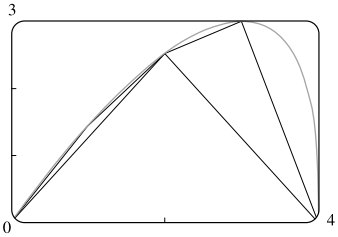
PROBLEMS PLUS ■ PAGE 534

1. About 1.85 inches from the center **3.** 0
7. $f(\pi) = -\pi/2$ **11.** $(b^b a^{-a})^{1/(b-a)}e^{-1}$ **13.** $\frac{1}{8}\pi - \frac{1}{12}$
15. $2 - \sin^{-1}\left(2/\sqrt{5}\right)$

CHAPTER 8

EXERCISES 8.1 ■ PAGE 543

1. $4\sqrt{5}$ **3.** 3.8202 **5.** 3.6095
7. $\frac{2}{243}\left(82\sqrt{82} - 1\right)$ **9.** $\frac{59}{24}$ **11.** $\frac{32}{3}$
13. $\ln\left(\sqrt{2}+1\right)$ **15.** $\frac{3}{4} + \frac{1}{2}\ln 2$ **17.** $\ln 3 - \frac{1}{2}$
19. $\sqrt{2} + \ln\left(1+\sqrt{2}\right)$ **21.** 10.0556
23. 15.374568 **25.** 7.118819
27. (a), (b)

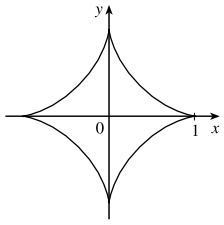


$L_1 = 4$,
$L_2 \approx 6.43$,
$L_4 \approx 7.50$

(c) $\int_0^4 \sqrt{1 + [4(3-x)/(3(4-x)^{2/3})]^2}\,dx$ (d) 7.7988

29. $\sqrt{5} - \ln\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right) - \sqrt{2} + \ln\left(1+\sqrt{2}\right)$

31. 6



33. $s(x) = \frac{2}{27}\left[(1 + 9x)^{3/2} - 10\sqrt{10}\right]$ **35.** $2\sqrt{2}\left(\sqrt{1 + x} - 1\right)$

37. 209.1 m **39.** 29.36 in. **41.** 12.4

EXERCISES 8.2 ■ PAGE 550

1. (a) (i) $\int_0^{\pi/3} 2\pi \tan x\sqrt{1 + \sec^4 x}\, dx$

(ii) $\int_0^{\pi/3} 2\pi x\sqrt{1 + \sec^4 x}\, dx$ (b) (i) 10.5017 (ii) 7.9353

3. (a) (i) $\int_{-1}^{1} 2\pi e^{-x^2}\sqrt{1 + 4x^2 e^{-2x^2}}\, dx$

(ii) $\int_0^1 2\pi x\sqrt{1 + 4x^2 e^{-2x^2}}\, dx$ (b) (i) 11.0753 (ii) 3.9603

5. $\frac{1}{27}\pi\left(145\sqrt{145} - 1\right)$ **7.** $\frac{98}{3}\pi$

9. $2\sqrt{1 + \pi^2} + (2/\pi)\ln\left(\pi + \sqrt{1 + \pi^2}\right)$ **11.** $\frac{21}{2}\pi$

13. $\frac{1}{27}\pi\left(145\sqrt{145} - 10\sqrt{10}\right)$ **15.** πa^2

17. 1,230,507 **19.** 24.144251

21. $\frac{1}{4}\pi\left[4\ln\left(\sqrt{17} + 4\right) - 4\ln\left(\sqrt{2} + 1\right) - \sqrt{17} + 4\sqrt{2}\right]$

23. $\frac{1}{6}\pi\left[\ln\left(\sqrt{10} + 3\right) + 3\sqrt{10}\right]$

27. (a) $\frac{1}{3}\pi a^2$ (b) $\frac{56}{45}\pi\sqrt{3}a^2$

29. (a) $2\pi\left[b^2 + \dfrac{a^2 b \sin^{-1}\left(\sqrt{a^2 - b^2}/a\right)}{\sqrt{a^2 - b^2}}\right]$

(b) $2\pi\left[a^2 + \dfrac{ab^2 \sin^{-1}\left(\sqrt{b^2 - a^2}/b\right)}{\sqrt{b^2 - a^2}}\right]$

31. $\int_a^b 2\pi[c - f(x)]\sqrt{1 + [f'(x)]^2}\, dx$ **33.** $4\pi^2 r^2$

EXERCISES 8.3 ■ PAGE 560

1. (a) 187.5 lb/ft^2 (b) 1875 lb (c) 562.5 lb

3. 6000 lb **5.** 6.7×10^4 N **7.** 9.8×10^3 N

9. 1.2×10^4 lb **11.** $\frac{2}{3}\delta a h^2$

13. 5.27×10^5 N **15.** (a) 314 N (b) 353 N

17. (a) 5.63×10^3 lb (b) 5.06×10^4 lb

(c) 4.88×10^4 lb (d) 3.03×10^5 lb

19. 4148 lb **21.** 330; 22 **23.** 10; 14; (1.4, 1) **25.** $\left(\frac{2}{3}, \frac{2}{3}\right)$

27. $\left(\dfrac{1}{e - 1}, \dfrac{e + 1}{4}\right)$ **29.** $\left(\frac{9}{20}, \frac{9}{20}\right)$

31. $\left(\dfrac{\pi\sqrt{2} - 4}{4\left(\sqrt{2} - 1\right)}, \dfrac{1}{4\left(\sqrt{2} - 1\right)}\right)$ **33.** $\left(\frac{8}{5}, -\frac{1}{2}\right)$

35. 60; 160; $\left(\frac{8}{3}, 1\right)$ **37.** $\left(-\frac{1}{5}, -\frac{12}{35}\right)$ **41.** $\left(0, \frac{1}{12}\right)$ **45.** $\frac{1}{3}\pi r^2 h$

EXERCISES 8.4 ■ PAGE 566

1. \$21,104 **3.** \$140,000; \$60,000 **5.** \$407.25

7. \$12,000 **9.** 3727; \$37,753

11. $\frac{2}{3}\left(16\sqrt{2} - 8\right) \approx$ \$9.75 million **13.** $\dfrac{(1 - k)(b^{2-k} - a^{2-k})}{(2 - k)(b^{1-k} - a^{1-k})}$

15. 1.19×10^{-4} cm^3/s **17.** 6.60 L/min **19.** 5.77 L/min

EXERCISES 8.5 ■ PAGE 573

1. (a) The probability that a randomly chosen tire will have a lifetime between 30,000 and 40,000 miles

(b) The probability that a randomly chosen tire will have a lifetime of at least 25,000 miles

3. (a) $f(x) \geqslant 0$ for all x and $\int_{-\infty}^{\infty} f(x)\, dx = 1$ (b) $\frac{17}{81}$

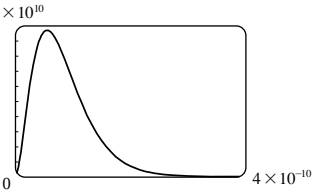
5. (a) $1/\pi$ (b) $\frac{1}{2}$

7. (a) $f(x) \geqslant 0$ for all x and $\int_{-\infty}^{\infty} f(x)\, dx = 1$ (b) 5

11. (a) $e^{-4/2.5} \approx 0.20$ (b) $1 - e^{-2/2.5} \approx 0.55$ (c) If you aren't served within 10 minutes, you get a free hamburger.

13. ≈44% **15.** (a) 0.0668 (b) ≈5.21% **17.** ≈0.9545

19. (b) 0; a_0 (c)



(d) $1 - 41e^{-8} \approx 0.986$ (e) $\frac{3}{2}a_0$

CHAPTER 8 REVIEW ■ PAGE 575

Exercises

1. $\frac{15}{2}$ **3.** (a) $\frac{21}{16}$ (b) $\frac{41}{10}\pi$

5. 3.8202 **7.** $\frac{124}{5}$ **9.** ≈458 lb **11.** $\left(\frac{8}{5}, 1\right)$

13. $\left(2, \frac{2}{3}\right)$ **15.** $2\pi^2$ **17.** \$7166.67

19. (a) $f(x) \geqslant 0$ for all x and $\int_{-\infty}^{\infty} f(x)\, dx = 1$

(b) ≈0.3455 (c) 5, yes

21. (a) $1 - e^{-3/8} \approx 0.31$ (b) $e^{-5/4} \approx 0.29$

(c) $8 \ln 2 \approx 5.55$ min

PROBLEMS PLUS ■ PAGE 577

1. $\frac{2}{3}\pi - \frac{1}{2}\sqrt{3}$

3. (a) $2\pi r(r \pm d)$ (b) $\approx 3.36 \times 10^6$ mi^2

(d) $\approx 7.84 \times 10^7$ mi^2

5. (a) $P(z) = P_0 + g\int_0^z \rho(x)\, dx$

(b) $(P_0 - \rho_0 gH)(\pi r^2) + \rho_0 gHe^{L/H}\int_{-r}^{r} e^{x/H} \cdot 2\sqrt{r^2 - x^2}\, dx$

7. Height $\sqrt{2}\, b$, volume $\left(\frac{28}{27}\sqrt{6} - 2\right)\pi b^3$ **9.** 0.14 m

11. $2/\pi$, $1/\pi$ **13.** $(0, -1)$

CHAPTER 9

EXERCISES 9.1 ■ PAGE 584

3. (a) $\frac{1}{2}$, -1 **5.** (d)

7. (a) It must be either 0 or decreasing

(c) $y = 0$ (d) $y = 1/(x + 2)$

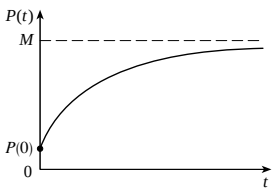
9. (a) $0 < P < 4200$ (b) $P > 4200$

(c) $P = 0$, $P = 4200$

13. (a) III (b) I (c) IV (d) II

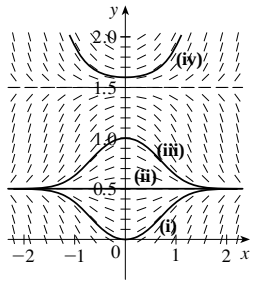
15. (a) At the beginning; stays positive, but decreases

(c)



EXERCISES 9.2 ■ PAGE 592

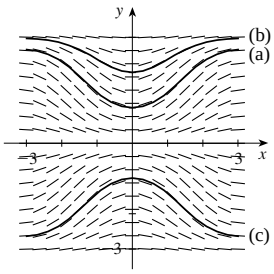
1. (a)



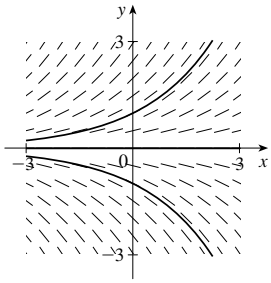
(b) $y = 0.5$, $y = 1.5$

3. III **5.** IV

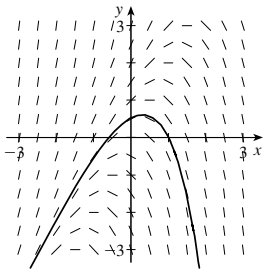
7.



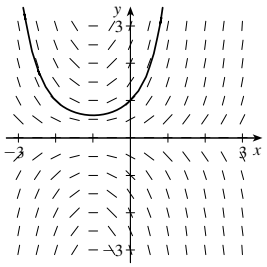
9.



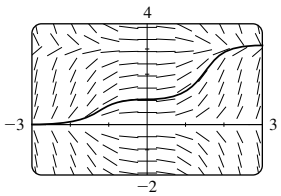
11.



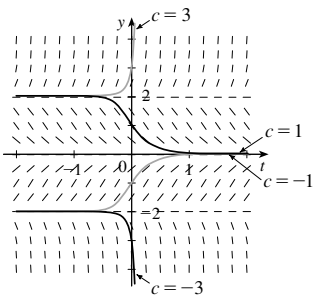
13.



15.



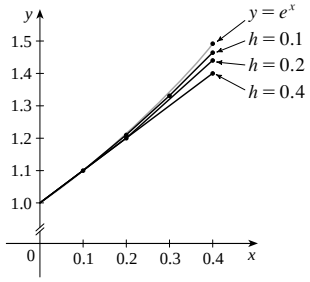
17.



$-2 \leqslant c \leqslant 2$; -2, 0, 2

19. (a) (i) 1.4 (ii) 1.44 (iii) 1.4641

(b)



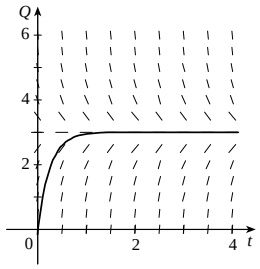
Underestimates

(c) (i) 0.0918 (ii) 0.0518 (iii) 0.0277
It appears that the error is also halved (approximately).

21. -1, -3, -6.5, -12.25 **23.** 1.7616

25. (a) (i) 3 (ii) 2.3928 (iii) 2.3701 (iv) 2.3681
(c) (i) -0.6321 (ii) -0.0249 (iii) -0.0022 (iv) -0.0002
It appears that the error is also divided by 10 (approximately).

27. (a), (d)



(b) 3
(c) Yes; $Q = 3$
(e) 2.77 C

EXERCISES 9.3 ■ PAGE 600

1. $y = \dfrac{2}{K - x^2}$, $y = 0$ **3.** $y = \sqrt[3]{3x + 3\ln|x| + K}$

5. $\frac{1}{2}y^2 - \cos y = \frac{1}{2}x^2 + \frac{1}{4}x^4 + C$

7. $e^y(y - 1) = C - \frac{1}{2}e^{-t^2}$ **9.** $p = Ke^{t^3/3 - t} - 1$

11. $y = -\sqrt{x^2 + 9}$ **13.** $u = -\sqrt{t^2 + \tan t + 25}$

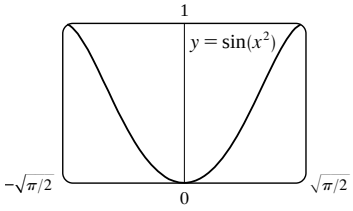
15. $\frac{1}{2}y^2 + \frac{1}{3}(3 + y^2)^{3/2} = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \frac{41}{12}$

17. $y = \dfrac{4a}{\sqrt{3}} \sin x - a$

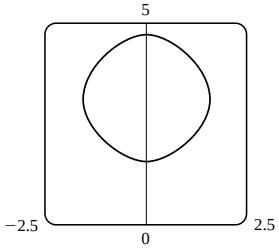
19. $y = e^{x^2/2}$ **21.** $y = Ke^x - x - 1$

23. (a) $\sin^{-1}y = x^2 + C$

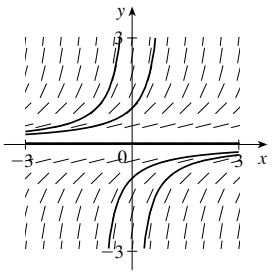
(b) $y = \sin(x^2)$, $-\sqrt{\pi/2} \le x \le \sqrt{\pi/2}$ (c) No



25. $\cos y = \cos x - 1$

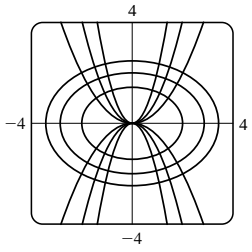


27. (a)

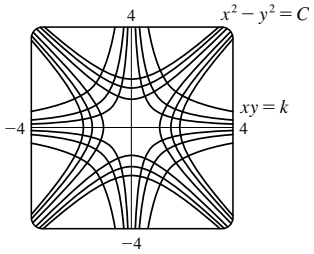


(b) $y = \dfrac{1}{K - x}$

29. $y = Cx^2$



31. $x^2 - y^2 = C$



33. $y = 1 + e^{2-x^2/2}$ **35.** $y = \left(\frac{1}{2}x^2 + 2\right)^2$

37. $Q(t) = 3 - 3e^{-4t}$; 3 **39.** $P(t) = M - Me^{-kt}$; M

41. (a) $x = a - \dfrac{4}{\left(kt + 2/\sqrt{a}\right)^2}$

(b) $t = \dfrac{2}{k\sqrt{a-b}}\left(\tan^{-1}\sqrt{\dfrac{b}{a-b}} - \tan^{-1}\sqrt{\dfrac{b-x}{a-b}}\right)$

43. (a) $C(t) = (C_0 - r/k)e^{-kt} + r/k$

(b) r/k; the concentration approaches r/k regardless of the value of C_0

45. (a) $15e^{-t/100}$ kg (b) $15e^{-0.2} \approx 12.3$ kg

47. About 4.9% **49.** g/k

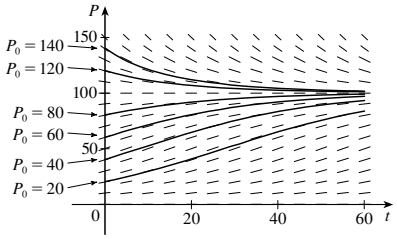
51. (a) $L_1 = KL_2^{\,k}$ (b) $B = KV^{0.0794}$

53. (a) $dA/dt = k\sqrt{A}\,(M - A)$ (b) $A(t) = M\left(\dfrac{Ce^{\sqrt{M}kt} - 1}{Ce^{\sqrt{M}kt} + 1}\right)^2$, where $C = \dfrac{\sqrt{M} + \sqrt{A_0}}{\sqrt{M} - \sqrt{A_0}}$ and $A_0 = A(0)$

EXERCISES 9.4 ■ PAGE 613

1. (a) 100; 0.05 (b) Where P is close to 0 or 100; on the line $P = 50$; $0 < P_0 < 100$; $P_0 > 100$

(c)



Solutions approach 100; some increase and some decrease, some have an inflection point but others don't; solutions with $P_0 = 20$ and $P_0 = 40$ have inflection points at $P = 50$

(d) $P = 0$, $P = 100$; other solutions move away from $P = 0$ and toward $P = 100$

3. (a) 3.23×10^7 kg (b) ≈1.55 years

5. 9000

7. (a) $dP/dt = \frac{1}{265}P(1 - P/100)$, P in billions

(b) 5.49 billion (c) In billions: 7.81, 27.72

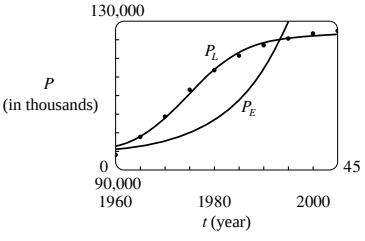
(d) In billions: 5.48, 7.61, 22.41

9. (a) $dy/dt = ky(1 - y)$ (b) $y = \dfrac{y_0}{y_0 + (1 - y_0)e^{-kt}}$

(c) 3:36 PM

13. $P_E(t) = 1578.3(1.0933)^t + 94{,}000$;

$P_L(t) = \dfrac{32{,}658.5}{1 + 12.75e^{-0.1706t}} + 94{,}000$

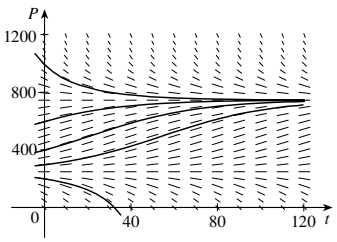


15. (a) $P(t) = \dfrac{m}{k} + \left(P_0 - \dfrac{m}{k}\right)e^{kt}$ (b) $m < kP_0$

(c) $m = kP_0$, $m > kP_0$ (d) Declining

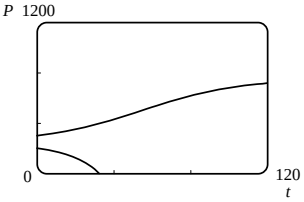
17. (a) Fish are caught at a rate of 15 per week.
(b) See part (d). (c) $P = 250$, $P = 750$

(d)

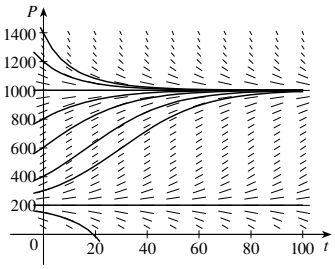


$0 < P_0 < 250$: $P \to 0$;
$P_0 = 250$: $P \to 250$;
$P_0 > 250$: $P \to 750$

(e) $P(t) = \dfrac{250 - 750ke^{t/25}}{1 - ke^{t/25}}$
where $k = \frac{1}{11}, -\frac{1}{9}$



19. (b)



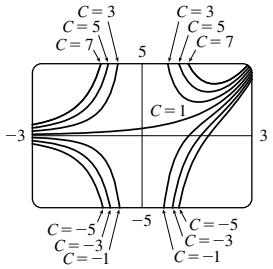
$0 < P_0 < 200$: $P \to 0$;
$P_0 = 200$: $P \to 200$;
$P_0 > 200$: $P \to 1000$

(c) $P(t) = \dfrac{m(M - P_0) + M(P_0 - m)e^{(M-m)(k/M)t}}{M - P_0 + (P_0 - m)e^{(M-m)(k/M)t}}$

21. (a) $P(t) = P_0 e^{(k/r)[\sin(rt - \phi) + \sin\phi]}$ (b) Does not exist

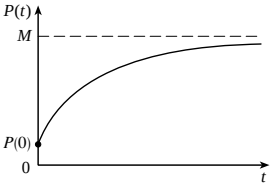
EXERCISES 9.5 ■ PAGE 620

1. Yes **3.** No **5.** $y = 1 + Ce^{-x}$
7. $y = x - 1 + Ce^{-x}$ **9.** $y = \frac{2}{3}\sqrt{x} + C/x$
11. $y = \dfrac{\int \sin(x^2)\,dx + C}{\sin x}$ **13.** $u = \dfrac{t^2 + 2t + 2C}{2(t + 1)}$
15. $y = \dfrac{1}{x}\ln x - \dfrac{1}{x} + \dfrac{3}{x^2}$ **17.** $u = -t^2 + t^3$
19. $y = -x\cos x - x$
21. $y = \dfrac{(x - 1)e^x + C}{x^2}$



25. $y = \pm\left(Cx^4 + \dfrac{2}{5x}\right)^{-1/2}$

27. (a) $I(t) = 4 - 4e^{-5t}$ (b) $4 - 4e^{-1/2} \approx 1.57$ A
29. $Q(t) = 3(1 - e^{-4t})$, $I(t) = 12e^{-4t}$
31. $P(t) = M + Ce^{-kt}$

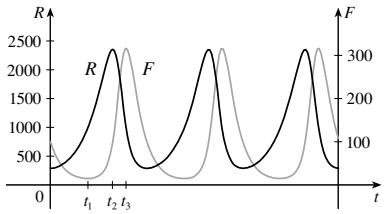


33. $y = \frac{2}{5}(100 + 2t) - 40{,}000(100 + 2t)^{-3/2}$; 0.2275 kg/L
35. (b) mg/c (c) $(mg/c)[t + (m/c)e^{-ct/m}] - m^2g/c^2$
37. (b) $P(t) = \dfrac{M}{1 + MCe^{-kt}}$

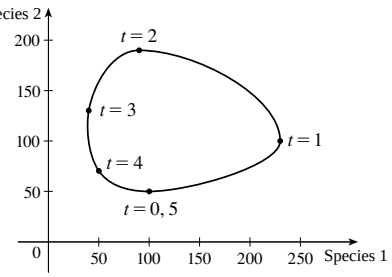
EXERCISES 9.6 ■ PAGE 627

1. (a) x = predators, y = prey; growth is restricted only by predators, which feed only on prey.
(b) x = prey, y = predators; growth is restricted by carrying capacity and by predators, which feed only on prey.
3. (a) Competition
(b) (i) $x = 0$, $y = 0$: zero populations
(ii) $x = 0$, $y = 400$: In the absence of an x-population, the y-population stabilizes at 400.
(iii) $x = 125$, $y = 0$: In the absence of a y-population, the x-population stabilizes at 125.
(iv) $x = 50$, $y = 300$: Both populations are stable.
5. (a) The rabbit population starts at about 300, increases to 2400, then decreases back to 300. The fox population starts at 100, decreases to about 20, increases to about 315, decreases to 100, and the cycle starts again.

(b)



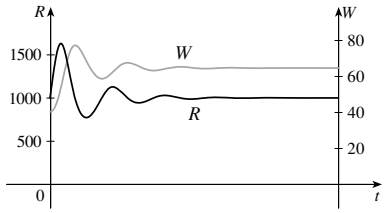
7.



11. (a) Population stabilizes at 5000.
(b) (i) $W = 0$, $R = 0$: Zero populations
(ii) $W = 0$, $R = 5000$: In the absence of wolves, the rabbit population is always 5000.
(iii) $W = 64$, $R = 1000$: Both populations are stable.

(c) The populations stabilize at 1000 rabbits and 64 wolves.

(d)



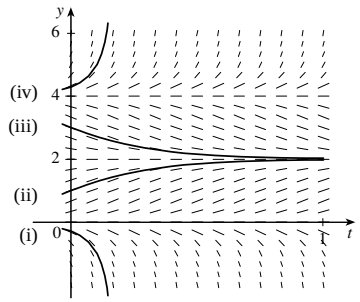
CHAPTER 9 REVIEW ■ PAGE 629

True-False Quiz

1. True **3.** False **5.** True **7.** True

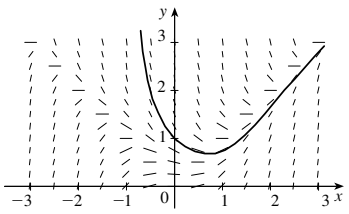
Exercises

1. (a)



(b) $0 \leq c \leq 4$; $y = 0$, $y = 2$, $y = 4$

3. (a)



$y(0.3) \approx 0.8$

(b) 0.75676

(c) $y = x$ and $y = -x$; there is a loc max or loc min

5. $y = \left(\frac{1}{2}x^2 + C\right)e^{-\sin x}$

7. $y = \pm\sqrt{\ln(x^2 + 2x^{3/2} + C)}$

9. $r(t) = 5e^{t-t^2}$ **11.** $y = \frac{1}{2}x(\ln x)^2 + 2x$ **13.** $x = C - \frac{1}{2}y^2$

15. (a) $P(t) = \dfrac{2000}{1 + 19e^{-0.1t}}$; ≈ 560 (b) $t = -10 \ln \frac{2}{57} \approx 33.5$

17. (a) $L(t) = L_\infty - [L_\infty - L(0)]e^{-kt}$ (b) $L(t) = 53 - 43e^{-0.2t}$

19. 15 days **21.** $k \ln h + h = (-R/V)t + C$

23. (a) Stabilizes at 200,000

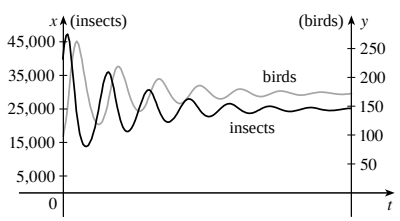
(b) (i) $x = 0$, $y = 0$: Zero populations

(ii) $x = 200{,}000$, $y = 0$: In the absence of birds, the insect population is always 200,000.

(iii) $x = 25{,}000$, $y = 175$: Both populations are stable.

(c) The populations stabilize at 25,000 insects and 175 birds.

(d)



25. (a) $y = (1/k)\cosh kx + a - 1/k$ or $y = (1/k)\cosh kx - (1/k)\cosh kb + h$ (b) $(2/k)\sinh kb$

PROBLEMS PLUS ■ PAGE 633

1. $f(x) = \pm 10e^x$ **5.** $y = x^{1/n}$ **7.** 20°C

9. (b) $f(x) = \dfrac{x^2 - L^2}{4L} - \frac{1}{2}L \ln\left(\dfrac{x}{L}\right)$ (c) No

11. (a) 9.8 h (b) $31{,}900\pi$ ft^2; 2000π ft^2/h

(c) 5.1 h

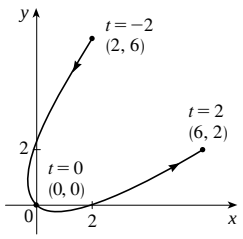
13. $x^2 + (y - 6)^2 = 25$

15. $y = K/x$, $K \neq 0$

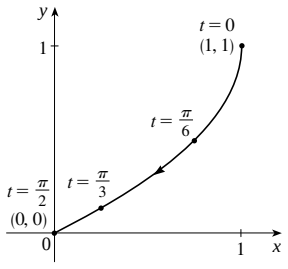
CHAPTER 10

EXERCISES 10.1 ■ PAGE 641

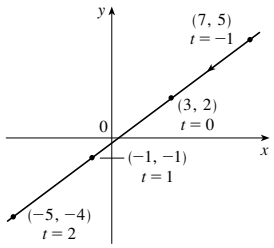
1.



3.

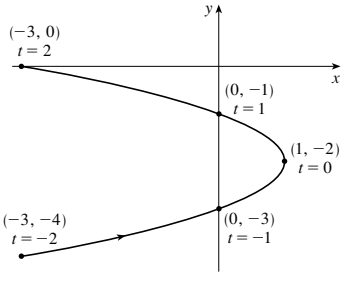


5. (a)



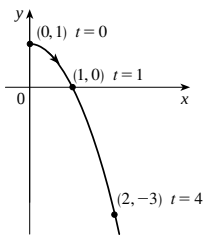
(b) $y = \frac{3}{4}x - \frac{1}{4}$

7. (a)



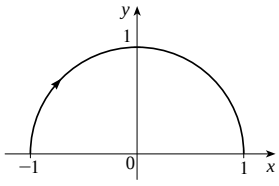
(b) $x = -(y + 2)^2 + 1$, $-4 \leq y \leq 0$

9. (a)

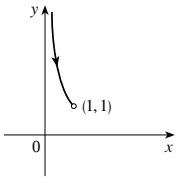


(b) $y = 1 - x^2,\ x \geqslant 0$

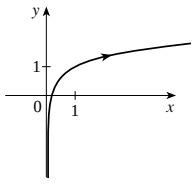
11. (a) $x^2 + y^2 = 1,\ y \geqslant 0$ (b)



13. (a) $y = 1/x,\ y > 1$ (b)

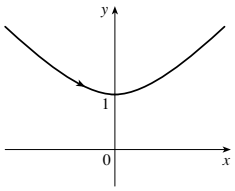


15. (a) $y = \frac{1}{2}\ln x + 1$ (b)



17. (a) $y^2 - x^2 = 1,\ y \geqslant 1$

(b)

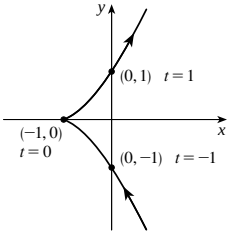


19. Moves counterclockwise along the circle $(x - 3)^2 + (y - 1)^2 = 4$ from (3, 3) to (3, −1)

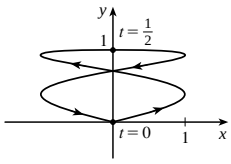
21. Moves 3 times clockwise around the ellipse $(x^2/25) + (y^2/4) = 1$, starting and ending at (0, −2)

23. It is contained in the rectangle described by $1 \leqslant x \leqslant 4$ and $2 \leqslant y \leqslant 3$.

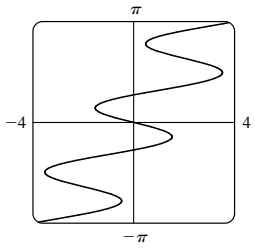
25.



27.



29.



31. (b) $x = -2 + 5t,\ y = 7 - 8t,\ 0 \leqslant t \leqslant 1$

33. (a) $x = 2\cos t,\ y = 1 - 2\sin t,\ 0 \leqslant t \leqslant 2\pi$

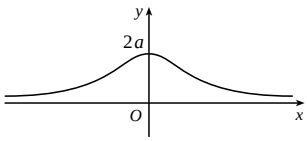
(b) $x = 2\cos t,\ y = 1 + 2\sin t,\ 0 \leqslant t \leqslant 6\pi$

(c) $x = 2\cos t,\ y = 1 + 2\sin t,\ \pi/2 \leqslant t \leqslant 3\pi/2$

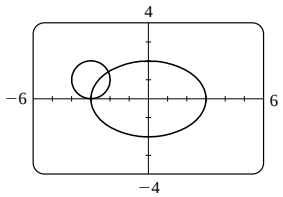
37. The curve $y = x^{2/3}$ is generated in (a). In (b), only the portion with $x \geqslant 0$ is generated, and in (c) we get only the portion with $x > 0$.

41. $x = a\cos\theta,\ y = b\sin\theta;\ (x^2/a^2) + (y^2/b^2) = 1$, ellipse

43.



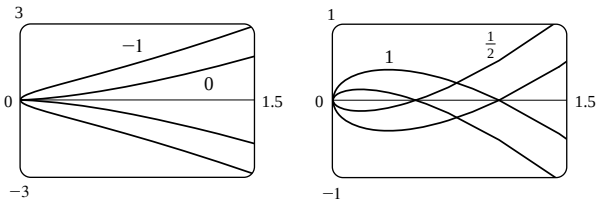
45. (a) Two points of intersection



(b) One collision point at $(-3, 0)$ when $t = 3\pi/2$

(c) There are still two intersection points, but no collision point.

47. For $c = 0$, there is a cusp; for $c > 0$, there is a loop whose size increases as c increases.



49. The curves roughly follow the line $y = x$, and they start having loops when a is between 1.4 and 1.6 (more precisely, when $a > \sqrt{2}$). The loops increase in size as a increases.

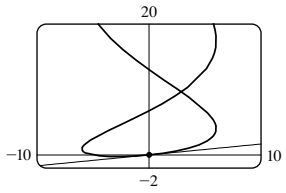
51. As n increases, the number of oscillations increases; a and b determine the width and height.

EXERCISES 10.2 ■ PAGE 651

1. $\dfrac{2t + 1}{t\cos t + \sin t}$ **3.** $y = -\frac{3}{2}x + 7$ **5.** $y = \pi x + \pi^2$

7. $y = 2x + 1$

9. $y = \frac{1}{6}x$



11. $\dfrac{2t+1}{2t}$, $-\dfrac{1}{4t^3}$, $t < 0$ **13.** $e^{-2t}(1-t)$, $e^{-3t}(2t-3)$, $t > \frac{3}{2}$

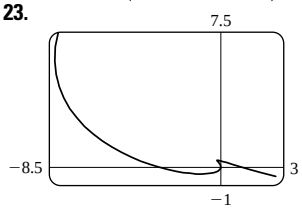
15. $-\frac{3}{2}\tan t$, $-\frac{3}{4}\sec^3 t$, $\pi/2 < t < 3\pi/2$

17. Horizontal at $(0, -3)$, vertical at $(\pm 2, -2)$

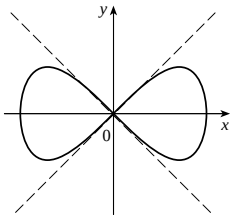
19. Horizontal at $\left(\frac{1}{2}, -1\right)$ and $\left(-\frac{1}{2}, 1\right)$, no vertical

21. $(0.6, 2)$; $\left(5 \cdot 6^{-6/5}, e^{6^{-1/5}}\right)$

23.



25. $y = x$, $y = -x$



27. (a) $d \sin\theta/(r - d\cos\theta)$ **29.** $\left(\frac{16}{27}, \frac{29}{9}\right)$, $(-2, -4)$

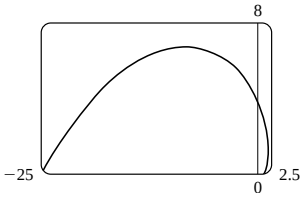
31. πab **33.** $3 - e$ **35.** $2\pi r^2 + \pi d^2$

37. $\int_0^2 \sqrt{2 + 2e^{-2t}}\, dt \approx 3.1416$

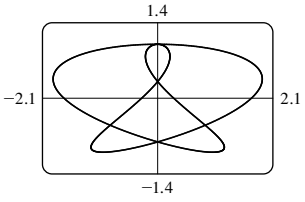
39. $\int_0^{4\pi} \sqrt{5 - 4\cos t}\, dt \approx 26.7298$ **41.** $4\sqrt{2} - 2$

43. $\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1 + \sqrt{2}\right)$

45. $\sqrt{2}\,(e^{\pi} - 1)$

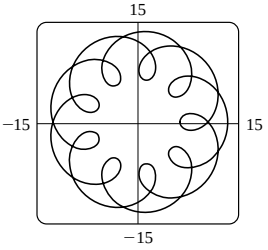


47. 16.7102



49. 612.3053 **51.** $6\sqrt{2}$, $\sqrt{2}$

55. (a)



$t \in [0, 4\pi]$

(b) 294

57. $\int_0^{\pi/2} 2\pi t \cos t \sqrt{t^2 + 1}\, dt \approx 4.7394$

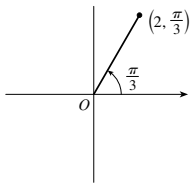
59. $\int_0^1 2\pi(t^2 + 1)e^t \sqrt{e^{2t}(t+1)^2 + (t^2 + 2t + 2)}\, dt \approx 103.5999$

61. $\frac{2}{1215}\pi\left(247\sqrt{13} + 64\right)$ **63.** $\frac{6}{5}\pi a^2$

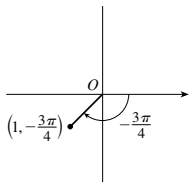
65. $\frac{24}{5}\pi\left(949\sqrt{26} + 1\right)$ **71.** $\frac{1}{4}$

EXERCISES 10.3 ■ PAGE 662

1. (a)

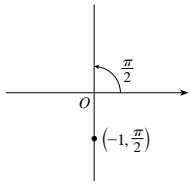


(b)



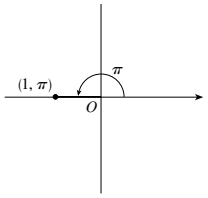
$(2, 7\pi/3)$, $(-2, 4\pi/3)$ $(1, 5\pi/4)$, $(-1, \pi/4)$

(c)

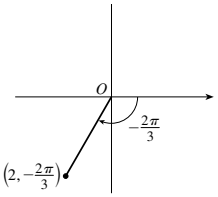


$(1, 3\pi/2)$, $(-1, 5\pi/2)$

3. (a)

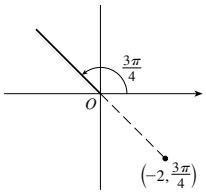


(b)



$(-1, 0)$ $\left(-1, -\sqrt{3}\right)$

(c)

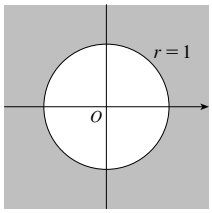


$\left(\sqrt{2}, -\sqrt{2}\right)$

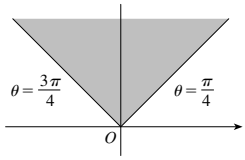
5. (a) (i) $\left(2\sqrt{2}, 7\pi/4\right)$ (ii) $\left(-2\sqrt{2}, 3\pi/4\right)$

(b) (i) $(2, 2\pi/3)$ (ii) $(-2, 5\pi/3)$

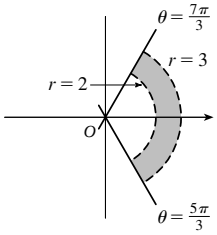
7.



9.



11.



13. $2\sqrt{3}$ **15.** Circle, center O, radius $\sqrt{5}$

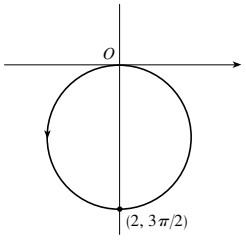
17. Circle, center $(1, 0)$, radius 1

19. Hyperbola, center O, foci on x-axis

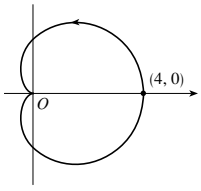
21. $r = 2 \csc\theta$ **23.** $r = 1/(\sin\theta - 3\cos\theta)$

25. $r = 2c\cos\theta$ **27.** (a) $\theta = \pi/6$ (b) $x = 3$

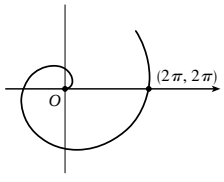
29.



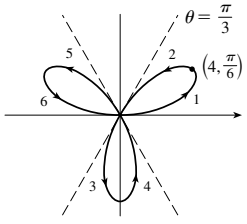
31.



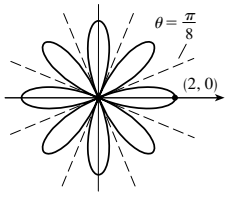
33.



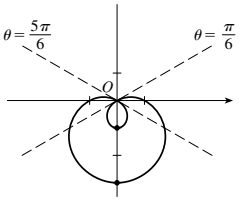
35.



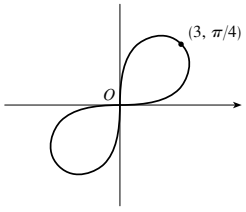
37.



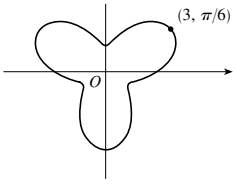
39.



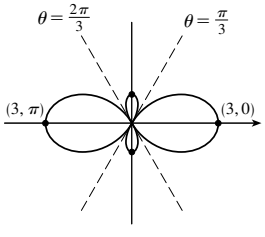
41.



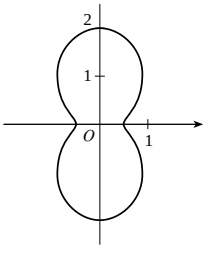
43.



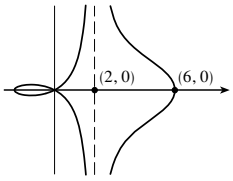
45.



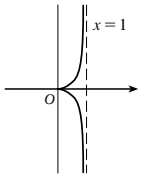
47.



49.



51.



53. (a) For $c < -1$, the inner loop begins at $\theta = \sin^{-1}(-1/c)$ and ends at $\theta = \pi - \sin^{-1}(-1/c)$; for $c > 1$, it begins at $\theta = \pi + \sin^{-1}(1/c)$ and ends at $\theta = 2\pi - \sin^{-1}(1/c)$.

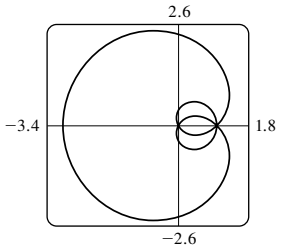
55. $\sqrt{3}$ **57.** $-\pi$ **59.** 1

61. Horizontal at $\left(3/\sqrt{2},\ \pi/4\right)$, $\left(-3/\sqrt{2},\ 3\pi/4\right)$; vertical at $(3, 0)$, $(0,\ \pi/2)$

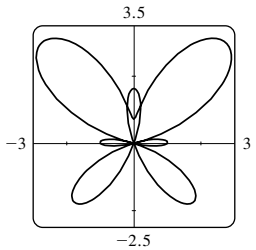
63. Horizontal at $\left(\frac{3}{2},\ \pi/3\right)$, $(0,\ \pi)$ [the pole], and $\left(\frac{3}{2},\ 5\pi/3\right)$; vertical at $(2, 0)$, $\left(\frac{1}{2},\ 2\pi/3\right)$, $\left(\frac{1}{2},\ 4\pi/3\right)$

65. Center $(b/2,\ a/2)$, radius $\sqrt{a^2 + b^2}/2$

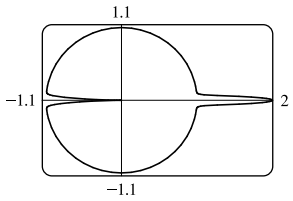
67.



69.



71.



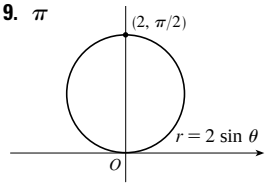
73. By counterclockwise rotation through angle $\pi/6$, $\pi/3$, or α about the origin

75. For $c = 0$, the curve is a circle. As c increases, the left side gets flatter, then has a dimple for $0.5 < c < 1$, a cusp for $c = 1$, and a loop for $c > 1$.

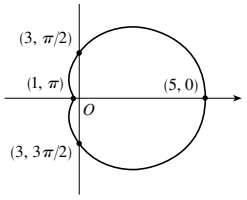
EXERCISES 10.4 ■ PAGE 668

1. $e^{-\pi/4} - e^{-\pi/2}$ **3.** $\frac{9}{2}$ **5.** π^2 **7.** $\frac{41}{4}\pi$

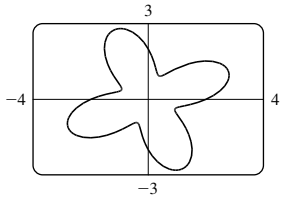
9. π



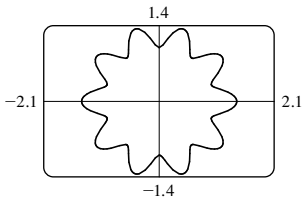
11. 11π



13. $\frac{9}{2}\pi$



15. $\frac{3}{2}\pi$



17. $\frac{4}{3}\pi$ **19.** $\frac{1}{16}\pi$ **21.** $\pi - \frac{3}{2}\sqrt{3}$ **23.** $\frac{1}{3}\pi + \frac{1}{2}\sqrt{3}$

25. $4\sqrt{3} - \frac{4}{3}\pi$ **27.** π **29.** $\frac{5}{24}\pi - \frac{1}{4}\sqrt{3}$ **31.** $\frac{1}{2}\pi - 1$

33. $1 - \frac{1}{2}\sqrt{2}$ **35.** $\frac{1}{4}\left(\pi + 3\sqrt{3}\right)$

37. $\left(\frac{3}{2}, \pi/6\right)$, $\left(\frac{3}{2}, 5\pi/6\right)$, and the pole

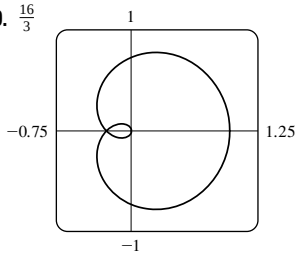
39. $(1, \theta)$ where $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$
and $(-1, \theta)$ where $\theta = 7\pi/12, 11\pi/12, 19\pi/12, 23\pi/12$

41. $\left(\frac{1}{2}\sqrt{3}, \pi/3\right)$, $\left(\frac{1}{2}\sqrt{3}, 2\pi/3\right)$, and the pole

43. Intersection at $\theta \approx 0.89, 2.25$; area ≈ 3.46

45. 2π **47.** $\frac{8}{3}[(\pi^2 + 1)^{3/2} - 1]$

49. $\frac{16}{3}$

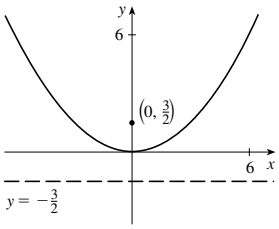


51. 2.4221 **53.** 8.0091

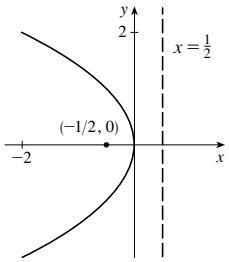
55. (b) $2\pi\left(2 - \sqrt{2}\right)$

EXERCISES 10.5 ■ PAGE 676

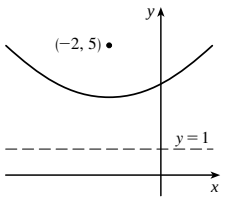
1. $(0, 0)$, $\left(0, \frac{3}{2}\right)$, $y = -\frac{3}{2}$



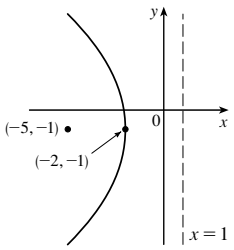
3. $(0, 0)$, $\left(-\frac{1}{2}, 0\right)$, $x = \frac{1}{2}$



5. $(-2, 3)$, $(-2, 5)$, $y = 1$

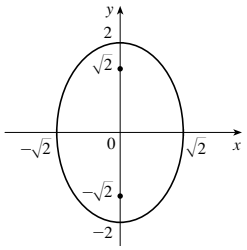


7. $(-2, -1)$, $(-5, -1)$, $x = 1$

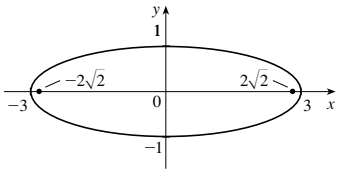


9. $x = -y^2$, focus $\left(-\frac{1}{4}, 0\right)$, directrix $x = \frac{1}{4}$

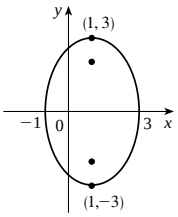
11. $(0, \pm 2)$, $\left(0, \pm\sqrt{2}\right)$



13. $(\pm 3, 0)$, $\left(\pm 2\sqrt{2}, 0\right)$

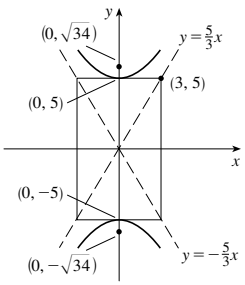


15. $(1, \pm 3)$, $\left(1, \pm\sqrt{5}\right)$

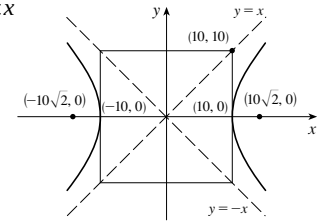


17. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, foci $\left(0, \pm\sqrt{5}\right)$

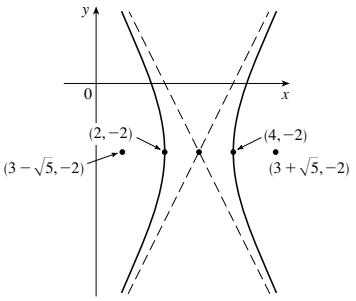
19. $(0, \pm 5)$; $\left(0, \pm\sqrt{34}\right)$; $y = \pm\frac{5}{3}x$



21. $(\pm 10, 0)$, $\left(\pm 10\sqrt{2}, 0\right)$, $y = \pm x$



23. $(4, -2)$, $(2, -2)$;
$\left(3 \pm \sqrt{5}, -2\right)$;
$y + 2 = \pm 2(x - 3)$



25. Parabola, $(0, -1)$, $\left(0, -\frac{3}{4}\right)$

27. Ellipse, $\left(\pm\sqrt{2}, 1\right)$, $(\pm 1, 1)$

29. Hyperbola, $(0, 1)$, $(0, -3)$; $\left(0, -1 \pm \sqrt{5}\right)$

31. $y^2 = 4x$ **33.** $y^2 = -12(x + 1)$ **35.** $y - 3 = 2(x - 2)^2$

37. $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$ **39.** $\dfrac{x^2}{12} + \dfrac{(y-4)^2}{16} = 1$

41. $\dfrac{(x+1)^2}{12} + \dfrac{(y-4)^2}{16} = 1$ **43.** $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

45. $\dfrac{(y-1)^2}{25} - \dfrac{(x+3)^2}{39} = 1$ **47.** $\dfrac{x^2}{9} - \dfrac{y^2}{36} = 1$

49. $\dfrac{x^2}{3{,}763{,}600} + \dfrac{y^2}{3{,}753{,}196} = 1$

51. (a) $\dfrac{121x^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1$ (b) ≈248 mi

55. (a) Ellipse (b) Hyperbola (c) No curve

59. 15.9

61. $\dfrac{b^2c}{a} + ab \ln\left(\dfrac{a}{b+c}\right)$ where $c^2 = a^2 + b^2$

63. $(0, 4/\pi)$

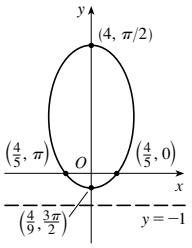
EXERCISES 10.6 ■ PAGE 684

1. $r = \dfrac{4}{2 + \cos\theta}$ **3.** $r = \dfrac{6}{2 + 3\sin\theta}$

5. $r = \dfrac{8}{1 - \sin\theta}$ **7.** $r = \dfrac{4}{2 + \cos\theta}$

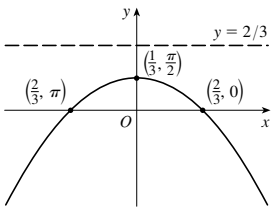
9. (a) $\frac{4}{5}$ (b) Ellipse (c) $y = -1$

(d)



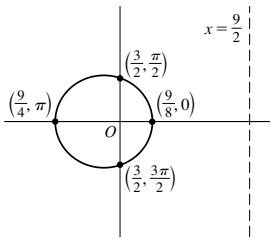
11. (a) 1 (b) Parabola (c) $y = \frac{2}{3}$

(d)



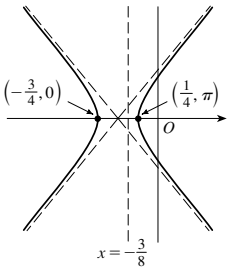
13. (a) $\frac{1}{3}$ (b) Ellipse (c) $x = \frac{9}{2}$

(d)

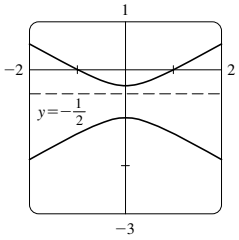


15. (a) 2 (b) Hyperbola (c) $x = -\frac{3}{8}$

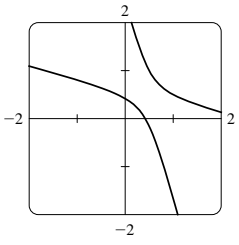
(d)



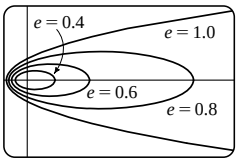
17. (a) $2, y = -\frac{1}{2}$



(b) $r = \dfrac{1}{1 - 2\sin(\theta - 3\pi/4)}$



19. The ellipse is nearly circular when e is close to 0 and becomes more elongated as $e \to 1^-$. At $e = 1$, the curve becomes a parabola.



25. $r = \dfrac{2.26 \times 10^8}{1 + 0.093\cos\theta}$

27. 35.64 AU **29.** 7.0×10^7 km **31.** 3.6×10^8 km

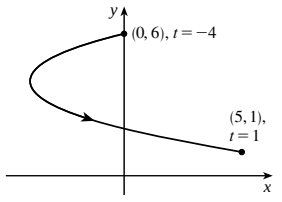
CHAPTER 10 REVIEW ■ PAGE 685

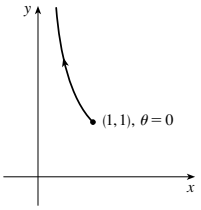
True-False Quiz

1. False **3.** False **5.** True **7.** False **9.** True

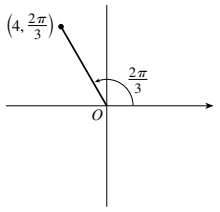
Exercises

1. $x = y^2 - 8y + 12$ **3.** $y = 1/x$



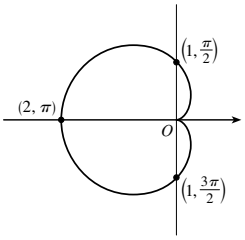


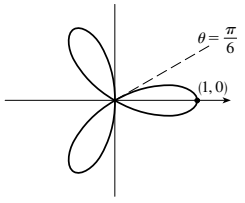
5. $x = t, y = \sqrt{t}$; $x = t^4, y = t^2$; $x = \tan^2 t, y = \tan t, 0 \leq t < \pi/2$

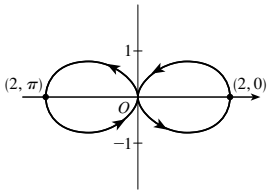
7. (a) 

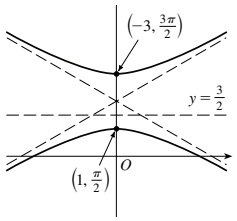
(b) $(3\sqrt{2}, 3\pi/4)$, $(-3\sqrt{2}, 7\pi/4)$

$(-2, 2\sqrt{3})$

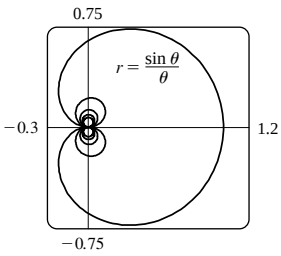
9. 

11. 

13. 

15. 

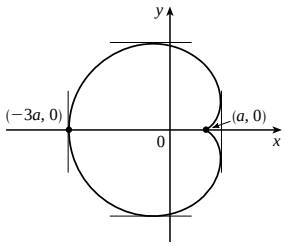
17. $r = \dfrac{2}{\cos\theta + \sin\theta}$

19. 

21. 2 **23.** −1

25. $\dfrac{1+\sin t}{1+\cos t}$, $\dfrac{1+\cos t+\sin t}{(1+\cos t)^3}$ **27.** $(\frac{11}{8}, \frac{3}{4})$

29. Vertical tangent at $(\frac{3}{2}a, \pm\frac{1}{2}\sqrt{3}\,a)$, $(-3a, 0)$; horizontal tangent at $(a, 0)$, $(-\frac{1}{2}a, \pm\frac{3}{2}\sqrt{3}a)$



31. 18 **33.** $(2, \pm\pi/3)$ **35.** $\frac{1}{2}(\pi - 1)$

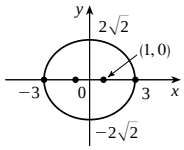
37. $2(5\sqrt{5} - 1)$

39. $\dfrac{2\sqrt{\pi^2+1} - \sqrt{4\pi^2+1}}{2\pi} + \ln\left(\dfrac{2\pi + \sqrt{4\pi^2+1}}{\pi + \sqrt{\pi^2+1}}\right)$

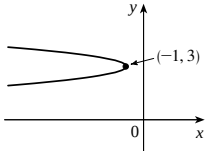
41. $471{,}295\pi/1024$

43. All curves have the vertical asymptote $x = 1$. For $c < -1$, the curve bulges to the right. At $c = -1$, the curve is the line $x = 1$. For $-1 < c < 0$, it bulges to the left. At $c = 0$ there is a cusp at $(0, 0)$. For $c > 0$, there is a loop.

45. $(\pm1, 0)$, $(\pm3, 0)$

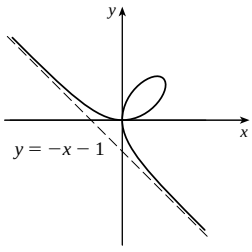


47. $(-\frac{25}{24}, 3)$, $(-1, 3)$



49. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ **51.** $\dfrac{y^2}{72/5} - \dfrac{x^2}{8/5} = 1$

53. $\dfrac{x^2}{25} + \dfrac{(8y-399)^2}{160{,}801} = 1$ **55.** $r = \dfrac{4}{3 + \cos\theta}$

57. (a) At (0, 0) and $(\frac{3}{2}, \frac{3}{2})$
(b) Horizontal tangents at (0, 0) and $(\sqrt[3]{2}, \sqrt[3]{4})$; vertical tangents at (0, 0) and $(\sqrt[3]{4}, \sqrt[3]{2})$
(d) 

(g) $\frac{3}{2}$

PROBLEMS PLUS ■ PAGE 688

1. $\ln(\pi/2)$ **3.** $\left[-\frac{3}{4}\sqrt{3}, \frac{3}{4}\sqrt{3}\right] \times [-1, 2]$

CHAPTER 11

EXERCISES 11.1 ■ PAGE 700

Abbreviations: C, convergent; D, divergent

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.
(b) The terms a_n approach 8 as n becomes large.
(c) The terms a_n become large as n becomes large.

3. $1, \frac{4}{5}, \frac{3}{5}, \frac{8}{17}, \frac{5}{13}$ **5.** $\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \frac{1}{3125}$ **7.** $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$

9. 1, 2, 7, 32, 157 **11.** $2, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}$ **13.** $a_n = 1/(2n-1)$

15. $a_n = -3\left(-\frac{2}{3}\right)^{n-1}$ **17.** $a_n = (-1)^{n+1}\dfrac{n^2}{n+1}$

19. 0.4286, 0.4615, 0.4737, 0.4800, 0.4839, 0.4865, 0.4884, 0.4898, 0.4909, 0.4918; yes; $\frac{1}{2}$

21. 0.5000, 1.2500, 0.8750, 1.0625, 0.9688, 1.0156, 0.9922, 1.0039, 0.9980, 1.0010; yes; 1

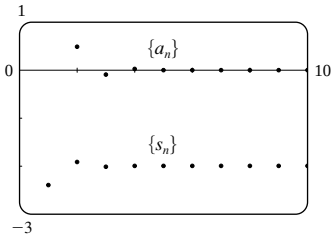
23. 1 **25.** 5 **27.** 1 **29.** 1 **31.** D **33.** 0
35. D **37.** 0 **39.** 0 **41.** 0 **43.** 0 **45.** 1
47. e^2 **49.** $\ln 2$ **51.** $\pi/2$ **53.** D **55.** D
57. 1 **59.** $\frac{1}{2}$ **61.** D **63.** 0
65. (a) 1060, 1123.60, 1191.02, 1262.48, 1338.23 (b) D

67. (a) $P_n = 1.08P_{n-1} - 300$ (b) 5734
69. $-1 < r < 1$
71. Convergent by the Monotonic Sequence Theorem; $5 \leq L < 8$
73. Decreasing; yes **75.** Not monotonic; no
77. Decreasing; yes
79. 2 **81.** $\frac{1}{2}(3 + \sqrt{5})$ **83.** (b) $\frac{1}{2}(1 + \sqrt{5})$
85. (a) 0 (b) 9, 11

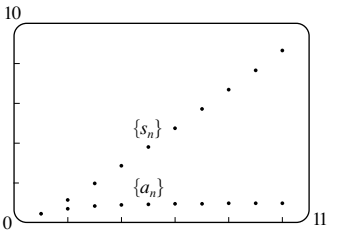
EXERCISES 11.2 ■ PAGE 711

1. (a) A sequence is an ordered list of numbers whereas a series is the *sum* of a list of numbers.
(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.
3. 2
5. 1, 1.125, 1.1620, 1.1777, 1.1857, 1.1903, 1.1932, 1.1952; C
7. 0.5, 1.3284, 2.4265, 3.7598, 5.3049, 7.0443, 8.9644, 11.0540; D

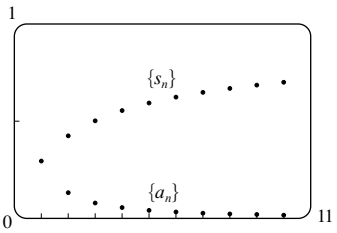
9. -2.40000, -1.92000, -2.01600, -1.99680, -2.00064, -1.99987, -2.00003, -1.99999, -2.00000, -2.00000; convergent, sum $= -2$



11. 0.44721, 1.15432, 1.98637, 2.88080, 3.80927, 4.75796, 5.71948, 6.68962, 7.66581, 8.64639; divergent



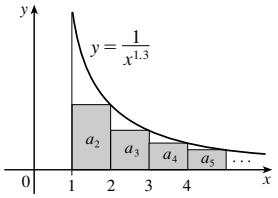
13. 0.29289, 0.42265, 0.50000, 0.55279, 0.59175, 0.62204, 0.64645, 0.66667, 0.68377, 0.69849; convergent, sum $= 1$



15. (a) C (b) D **17.** D **19.** $\frac{25}{3}$ **21.** 60 **23.** $\frac{1}{7}$
25. D **27.** D **29.** D **31.** $\frac{5}{2}$ **33.** D **35.** D
37. D **39.** D **41.** $e/(e-1)$ **43.** $\frac{3}{2}$ **45.** $\frac{11}{6}$ **47.** $e - 1$
49. (b) 1 (c) 2 (d) All rational numbers with a terminating decimal representation, except 0.
51. $\frac{8}{9}$ **53.** $\frac{838}{333}$ **55.** $5063/3300$
57. $-\frac{1}{5} < x < \frac{1}{5}$; $\frac{-5x}{1+5x}$ **59.** $-1 < x < 5$; $\frac{3}{5-x}$
61. $x > 2$ or $x < -2$; $\frac{x}{x-2}$ **63.** $x < 0$; $\frac{1}{1-e^x}$
65. 1 **67.** $a_1 = 0$, $a_n = \frac{2}{n(n+1)}$ for $n > 1$, sum $= 1$
69. (a) 157.875 mg; $\frac{3000}{19}(1 - 0.05^n)$ (b) 157.895 mg
71. (a) $S_n = \frac{D(1-c^n)}{1-c}$ (b) 5 **73.** $\frac{1}{2}(\sqrt{3} - 1)$
77. $\frac{1}{n(n+1)}$ **79.** The series is divergent.
85. $\{s_n\}$ is bounded and increasing.
87. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$
89. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}, \frac{(n+1)! - 1}{(n+1)!}$ (c) 1

EXERCISES 11.3 ■ PAGE 720

1. C



3. D **5.** C **7.** D **9.** C **11.** C **13.** D
15. C **17.** C **19.** C **21.** D **23.** C **25.** C
27. f is neither positive nor decreasing.
29. $p > 1$ **31.** $p < -1$ **33.** $(1, \infty)$
35. (a) $\frac{9}{10}\pi^4$ (b) $\frac{1}{90}\pi^4 - \frac{17}{16}$
37. (a) 1.54977, error ≤ 0.1 (b) 1.64522, error ≤ 0.005
(c) 1.64522 compared to 1.64493 (d) $n > 1000$
39. 0.00145 **45.** $b < 1/e$

EXERCISES 11.4 ■ PAGE 726

1. (a) Nothing (b) C **3.** C **5.** D **7.** C **9.** D
11. C **13.** C **15.** D **17.** D **19.** D **21.** C
23. C **25.** D **27.** C **29.** C **31.** D
33. 1.249, error < 0.1 **35.** 0.0739, error $< 6.4 \times 10^{-8}$
45. Yes

EXERCISES 11.5 ■ PAGE 731

1. (a) A series whose terms are alternately positive and negative (b) $0 < b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, where $b_n = |a_n|$ (c) $|R_n| \leq b_{n+1}$
3. C **5.** C **7.** D **9.** C **11.** C **13.** D **15.** C
17. C **19.** D **21.** -0.5507 **23.** 5 **25.** 4
27. -0.4597 **29.** 0.0676 **31.** An underestimate
33. p is not a negative integer **35.** $\{b_n\}$ is not decreasing

EXERCISES 11.6 ■ PAGE 737

Abbreviations: AC, absolutely convergent; CC, conditionally convergent

1. (a) D (b) C (c) May converge or diverge
3. AC **5.** CC **7.** AC **9.** D **11.** AC **13.** AC
15. AC **17.** CC **19.** AC **21.** AC **23.** D **25.** AC
27. AC **29.** D **31.** D **33.** AC

35. (a) and (d)

39. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521

(b) $n \geqslant 11$, 0.693109

45. (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$; $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

EXERCISES 11.7 ■ PAGE 740

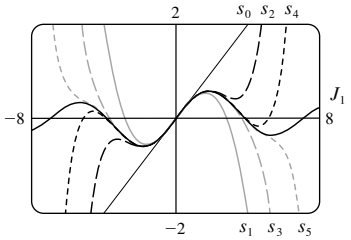
1. C **3.** D **5.** C **7.** D **9.** C **11.** C
13. C **15.** C **17.** C **19.** C **21.** D **23.** D
25. C **27.** C **29.** C **31.** D
33. C **35.** D **37.** C

EXERCISES 11.8 ■ PAGE 745

1. A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$, where x is a variable and a and the c_n's are constants

3. 1, $(-1, 1)$ **5.** 1, $[-1, 1)$
7. ∞, $(-\infty, \infty)$ **9.** 2, $(-2, 2)$ **11.** $\frac{1}{3}$, $\left[-\frac{1}{3}, \frac{1}{3}\right]$
13. 4, $(-4, 4]$ **15.** 1, $[1, 3]$ **17.** $\frac{1}{3}$, $\left[-\frac{13}{3}, -\frac{11}{3}\right)$
19. ∞, $(-\infty, \infty)$ **21.** b, $(a-b, a+b)$ **23.** 0, $\left\{\frac{1}{2}\right\}$
25. $\frac{1}{5}$, $\left[\frac{3}{5}, 1\right]$ **27.** ∞, $(-\infty, \infty)$ **29.** (a) Yes (b) No
31. k^k **33.** No
35. (a) $(-\infty, \infty)$

(b), (c)



37. $(-1, 1)$, $f(x) = (1+2x)/(1-x^2)$ **41.** 2

EXERCISES 11.9 ■ PAGE 751

1. 10 **3.** $\sum_{n=0}^{\infty} (-1)^n x^n$, $(-1, 1)$ **5.** $2\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n$, $(-3, 3)$

7. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}$, $(-3, 3)$ **9.** $1 + 2\sum_{n=1}^{\infty} x^n$, $(-1, 1)$

11. $\sum_{n=0}^{\infty} \left[(-1)^{n+1} - \frac{1}{2^{n+1}}\right] x^n$, $(-1, 1)$

13. (a) $\sum_{n=0}^{\infty} (-1)^n (n+1) x^n$, $R = 1$

(b) $\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) x^n$, $R = 1$

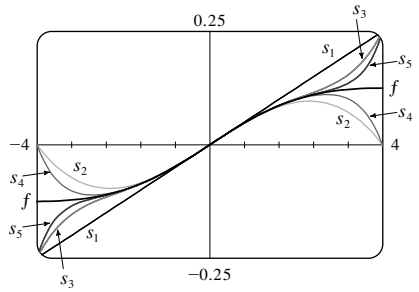
(c) $\frac{1}{2}\sum_{n=2}^{\infty} (-1)^n n(n-1) x^n$, $R = 1$

15. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}$, $R = 5$

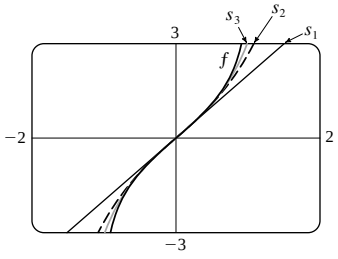
17. $\sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}$, $R = \frac{1}{4}$

19. $\sum_{n=0}^{\infty} (2n+1) x^n$, $R = 1$

21. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{16^{n+1}} x^{2n+1}$, $R = 4$



23. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}$, $R = 1$



25. $C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}$, $R = 1$

27. $C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+3}}{n(n+3)}$, $R = 1$

29. 0.199989 **31.** 0.000983 **33.** 0.19740
35. (b) 0.920 **39.** $[-1, 1]$, $[-1, 1)$, $(-1, 1)$

EXERCISES 11.10 ■ PAGE 765

1. $b_8 = f^{(8)}(5)/8!$ **3.** $\sum_{n=0}^{\infty} (n+1) x^n$, $R = 1$

5. $\sum_{n=0}^{\infty} (n+1) x^n$, $R = 1$

7. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}$, $R = \infty$

9. $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n$, $R = \infty$ **11.** $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$, $R = \infty$

13. $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4$, $R = \infty$

15. $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n} (x-2)^n$, $R = 2$

17. $\sum_{n=0}^{\infty} \frac{2^n e^6}{n!} (x-3)^n$, $R = \infty$

19. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{(2n)!} (x-\pi)^{2n}$, $R = \infty$

25. $1 - \frac{1}{4}x - \sum_{n=2}^{\infty} \frac{3 \cdot 7 \cdot \cdots \cdot (4n-5)}{4^n \cdot n!} x^n$, $R = 1$

27. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n$, $R = 2$

29. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}$, $R = \infty$

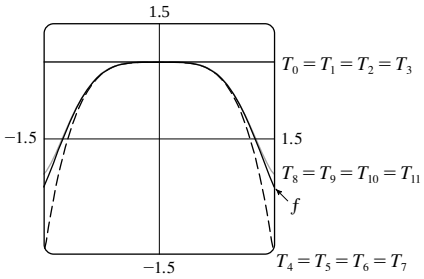
31. $\sum_{n=0}^{\infty} \frac{2^n + 1}{n!} x^n$, $R = \infty$

33. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n}(2n)!} x^{4n+1}$, $R = \infty$

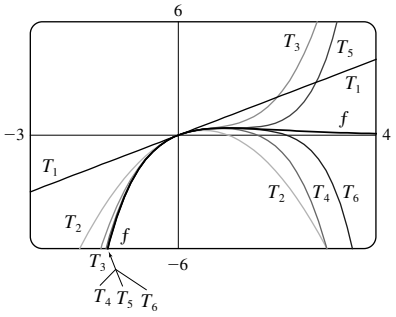
35. $\frac{1}{2}x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n!\, 2^{3n+1}} x^{2n+1}$, $R = 2$

37. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}}{(2n)!} x^{2n}$, $R = \infty$

39. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}$, $R = \infty$



41. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} x^n$, $R = \infty$



43. 0.99619

45. (a) $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n n!} x^{2n}$

(b) $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{(2n+1)2^n n!} x^{2n+1}$

47. $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!}$, $R = \infty$

49. $C + \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n\,(2n)!} x^{2n}$, $R = \infty$

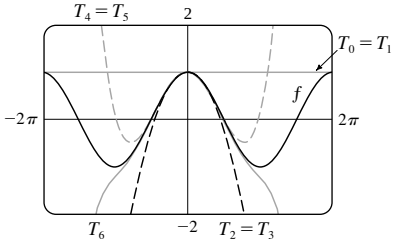
51. 0.0059 **53.** 0.40102 **55.** $\frac{1}{2}$ **57.** $\frac{1}{120}$

59. $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4$ **61.** $1 + \frac{1}{6}x^2 + \frac{7}{360}x^4$ **63.** e^{-x^4}

65. $\ln \frac{8}{5}$ **67.** $1/\sqrt{2}$ **69.** $e^3 - 1$

EXERCISES 11.11 ■ PAGE 774

1. (a) $T_0(x) = 1 = T_1(x)$, $T_2(x) = 1 - \frac{1}{2}x^2 = T_3(x)$,
$T_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 = T_5(x)$,
$T_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

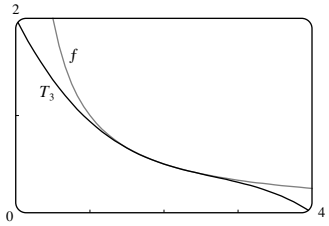


(b)

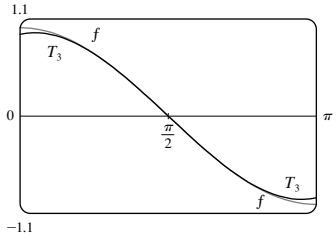
x	f	$T_0 = T_1$	$T_2 = T_3$	$T_4 = T_5$	T_6
$\frac{\pi}{4}$	0.7071	1	0.6916	0.7074	0.7071
$\frac{\pi}{2}$	0	1	−0.2337	0.0200	−0.0009
π	−1	1	−3.9348	0.1239	−1.2114

(c) As n increases, $T_n(x)$ is a good approximation to $f(x)$ on a larger and larger interval.

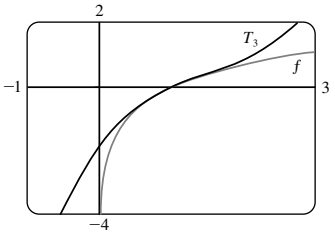
3. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$



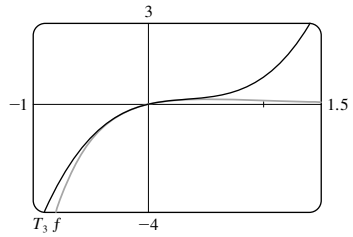
5. $-\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$



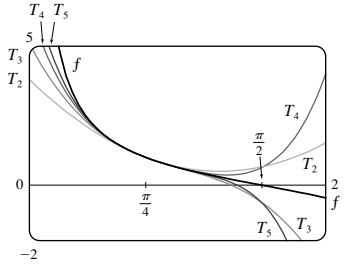
7. $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$



9. $x - 2x^2 + 2x^3$



11. $T_5(x) = 1 - 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 - \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{15}\left(x - \frac{\pi}{4}\right)^5$



13. (a) $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$ (b) 1.5625×10^{-5}

15. (a) $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$ (b) 0.000097

17. (a) $1 + \frac{1}{2}x^2$ (b) 0.0014

19. (a) $1 + x^2$ (b) 0.00006 **21.** (a) $x^2 - \frac{1}{6}x^4$ (b) 0.042

23. 0.17365 **25.** Four **27.** $-1.037 < x < 1.037$

29. $-0.86 < x < 0.86$ **31.** 21 m, no

37. (c) They differ by about 8×10^{-9} km.

CHAPTER 11 REVIEW ■ PAGE 778

True-False Quiz

1. False **3.** True **5.** False **7.** False **9.** False
11. True **13.** True **15.** False **17.** True
19. True **21.** True

Exercises

1. $\frac{1}{2}$ **3.** D **5.** 0 **7.** e^{12} **9.** 2 **11.** C **13.** C
15. D **17.** C **19.** C **21.** C **23.** CC **25.** AC

27. $\frac{1}{11}$ **29.** $\pi/4$ **31.** e^{-e} **35.** 0.9721

37. 0.18976224, error $< 6.4 \times 10^{-7}$

41. 4, $[-6, 2)$ **43.** 0.5, [2.5, 3.5)

45. $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{(2n)!}\left(x - \frac{\pi}{6}\right)^{2n} + \frac{\sqrt{3}}{(2n+1)!}\left(x - \frac{\pi}{6}\right)^{2n+1}\right]$

47. $\sum_{n=0}^{\infty}(-1)^n x^{n+2}$, $R = 1$ **49.** $\ln 4 - \sum_{n=1}^{\infty}\frac{x^n}{n4^n}$, $R = 4$

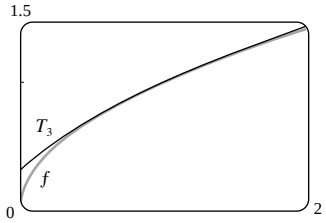
51. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{8n+4}}{(2n+1)!}$, $R = \infty$

53. $\frac{1}{2} + \sum_{n=1}^{\infty}\frac{1 \cdot 5 \cdot 9 \cdot \cdots \cdot (4n-3)}{n!\,2^{6n+1}}x^n$, $R = 16$

55. $C + \ln|x| + \sum_{n=1}^{\infty}\frac{x^n}{n \cdot n!}$

57. (a) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b)



(c) 0.000006

59. $-\frac{1}{6}$

PROBLEMS PLUS ■ PAGE 781

1. $15!/5! = 10{,}897{,}286{,}400$

3. (b) 0 if $x = 0$, $(1/x) - \cot x$ if $x \neq k\pi$, k an integer

5. (a) $s_n = 3 \cdot 4^n$, $l_n = 1/3^n$, $p_n = 4^n/3^{n-1}$ (c) $\frac{2}{5}\sqrt{3}$

9. $(-1, 1)$, $\frac{x^3 + 4x^2 + x}{(1-x)^4}$

11. $\ln\frac{1}{2}$ **13.** (a) $\frac{250}{101}\pi(e^{-(n-1)\pi/5} - e^{-n\pi/5})$ (b) $\frac{250}{101}\pi$

19. $\frac{\pi}{2\sqrt{3}} - 1$

21. $-\left(\frac{\pi}{2} - \pi k\right)^2$ where k is a positive integer

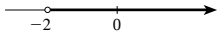
APPENDIXES

EXERCISES A ■ PAGE A9

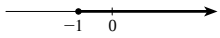
1. 18 **3.** π **5.** $5 - \sqrt{5}$ **7.** $2 - x$

9. $|x+1| = \begin{cases} x+1 & \text{for } x \geqslant -1 \\ -x-1 & \text{for } x < -1 \end{cases}$ **11.** $x^2 + 1$

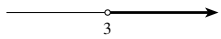
13. $(-2, \infty)$



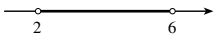
15. $[-1, \infty)$



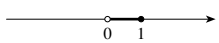
17. $(3, \infty)$



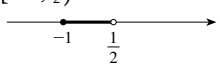
19. $(2, 6)$



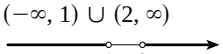
21. $(0, 1]$



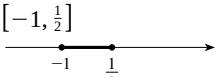
23. $\left[-1, \frac{1}{2}\right)$



25. $(-\infty, 1) \cup (2, \infty)$

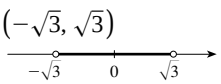


27. $\left[-1, \frac{1}{2}\right]$

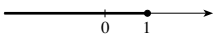


29. $(-\infty, \infty)$

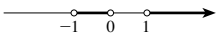
31. $\left(-\sqrt{3}, \sqrt{3}\right)$



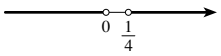
33. $(-\infty, 1]$



35. $(-1, 0) \cup (1, \infty)$



37. $(-\infty, 0) \cup \left(\frac{1}{4}, \infty\right)$

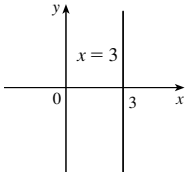


39. $10 \leqslant C \leqslant 35$ **41.** (a) $T = 20 - 10h$, $0 \leqslant h \leqslant 12$
(b) $-30°\text{C} \leqslant T \leqslant 20°\text{C}$ **43.** $\pm\frac{3}{2}$ **45.** $2, -\frac{4}{3}$
47. $(-3, 3)$ **49.** $(3, 5)$ **51.** $(-\infty, -7] \cup [-3, \infty)$
53. $[1.3, 1.7]$ **55.** $[-4, -1] \cup [1, 4]$
57. $x \geqslant (a + b)c/(ab)$ **59.** $x > (c - b)/a$

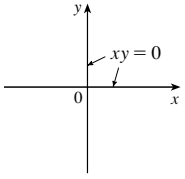
EXERCISES B ■ PAGE A15

1. 5 **3.** $\sqrt{74}$ **5.** $2\sqrt{37}$ **7.** 2 **9.** $-\frac{9}{2}$

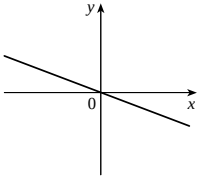
17.

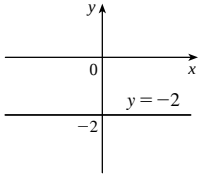


19.

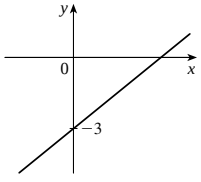


21. $y = 6x - 15$ **23.** $2x - 3y + 19 = 0$
25. $5x + y = 11$ **27.** $y = 3x - 2$ **29.** $y = 3x - 3$
31. $y = 5$ **33.** $x + 2y + 11 = 0$ **35.** $5x - 2y + 1 = 0$
37. $m = -\frac{1}{3}$, $b = 0$ **39.** $m = 0$, $b = -2$

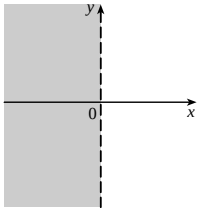




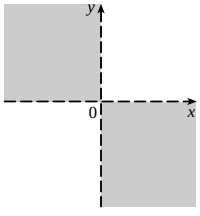
41. $m = \frac{3}{4}$, $b = -3$



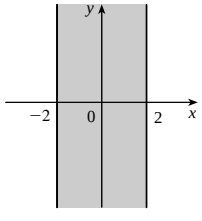
43.



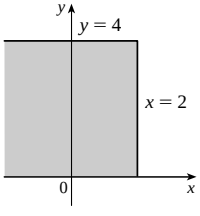
45.



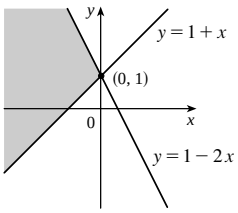
47.



49.



51.

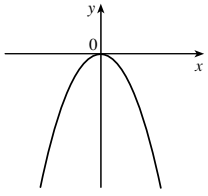


53. $(0, -4)$ **55.** (a) $(4, 9)$ (b) $(3.5, -3)$ **57.** $(1, -2)$
59. $y = x - 3$ **61.** (b) $4x - 3y - 24 = 0$

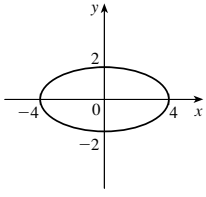
EXERCISES C ■ PAGE A23

1. $(x - 3)^2 + (y + 1)^2 = 25$ **3.** $x^2 + y^2 = 65$
5. $(2, -5)$, 4 **7.** $\left(-\frac{1}{2}, 0\right)$, $\frac{1}{2}$ **9.** $\left(\frac{1}{4}, -\frac{1}{4}\right)$, $\sqrt{10}/4$

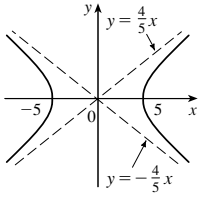
11. Parabola



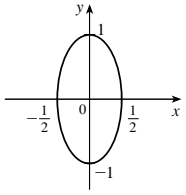
13. Ellipse



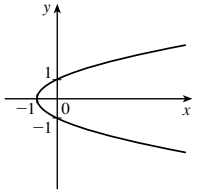
15. Hyperbola



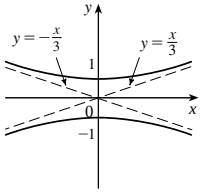
17. Ellipse



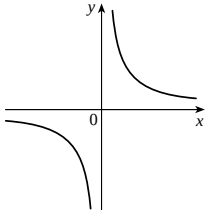
19. Parabola



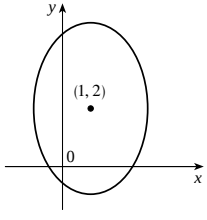
21. Hyperbola



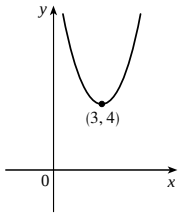
23. Hyperbola



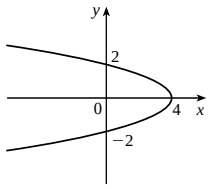
25. Ellipse



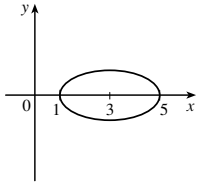
27. Parabola



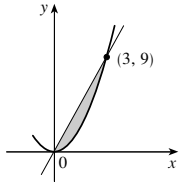
29. Parabola



31. Ellipse

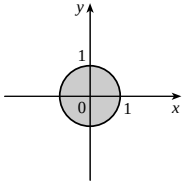


33.

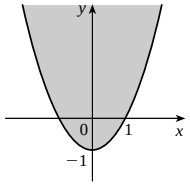


35. $y = x^2 - 2x$

37.



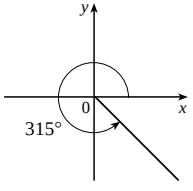
39.



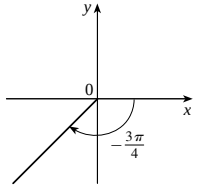
EXERCISES D ■ PAGE A32

1. $7\pi/6$ **3.** $\pi/20$ **5.** 5π **7.** $720°$ **9.** $75°$
11. $-67.5°$ **13.** 3π cm **15.** $\frac{2}{3}$ rad $= (120/\pi)°$

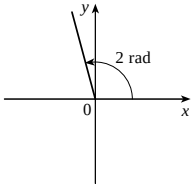
17.



19.



21.



23. $\sin(3\pi/4) = 1/\sqrt{2}$, $\cos(3\pi/4) = -1/\sqrt{2}$, $\tan(3\pi/4) = -1$, $\csc(3\pi/4) = \sqrt{2}$, $\sec(3\pi/4) = -\sqrt{2}$, $\cot(3\pi/4) = -1$

25. $\sin(9\pi/2) = 1$, $\cos(9\pi/2) = 0$, $\csc(9\pi/2) = 1$, $\cot(9\pi/2) = 0$, $\tan(9\pi/2)$ and $\sec(9\pi/2)$ undefined

27. $\sin(5\pi/6) = \frac{1}{2}$, $\cos(5\pi/6) = -\sqrt{3}/2$, $\tan(5\pi/6) = -1/\sqrt{3}$, $\csc(5\pi/6) = 2$, $\sec(5\pi/6) = -2/\sqrt{3}$, $\cot(5\pi/6) = -\sqrt{3}$

29. $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$

31. $\sin\phi = \sqrt{5}/3$, $\cos\phi = -\frac{2}{3}$, $\tan\phi = -\sqrt{5}/2$, $\csc\phi = 3/\sqrt{5}$, $\cot\phi = -2/\sqrt{5}$

33. $\sin\beta = -1/\sqrt{10}$, $\cos\beta = -3/\sqrt{10}$, $\tan\beta = \frac{1}{3}$, $\csc\beta = -\sqrt{10}$, $\sec\beta = -\sqrt{10}/3$

35. 5.73576 cm **37.** 24.62147 cm **59.** $\frac{1}{15}(4 + 6\sqrt{2})$

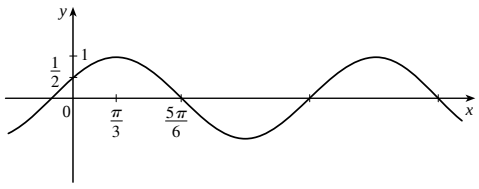
61. $\frac{1}{15}(3 + 8\sqrt{2})$ **63.** $\frac{24}{25}$ **65.** $\pi/3, 5\pi/3$

67. $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ **69.** $\pi/6, \pi/2, 5\pi/6, 3\pi/2$

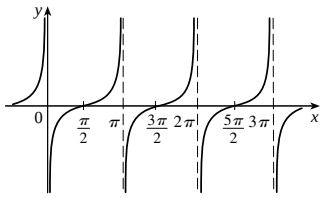
71. $0, \pi, 2\pi$ **73.** $0 \leqslant x \leqslant \pi/6$ and $5\pi/6 \leqslant x \leqslant 2\pi$

75. $0 \leqslant x < \pi/4$, $3\pi/4 < x < 5\pi/4$, $7\pi/4 < x \leqslant 2\pi$

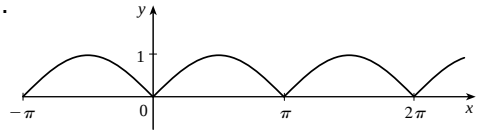
77.



79.



81.



89. 14.34457 cm^2

EXERCISES E ■ PAGE A38

1. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$ **3.** $3^4 + 3^5 + 3^6$

5. $-1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$ **7.** $1^{10} + 2^{10} + 3^{10} + \cdots + n^{10}$

9. $1 - 1 + 1 - 1 + \cdots + (-1)^{n-1}$ **11.** $\sum_{i=1}^{10} i$

13. $\sum_{i=1}^{19} \frac{i}{i+1}$ **15.** $\sum_{i=1}^{n} 2i$ **17.** $\sum_{i=0}^{5} 2^i$ **19.** $\sum_{i=1}^{n} x^i$

21. 80 **23.** 3276 **25.** 0 **27.** 61 **29.** $n(n+1)$

31. $n(n^2 + 6n + 17)/3$ **33.** $n(n^2 + 6n + 11)/3$

35. $n(n^3 + 2n^2 - n - 10)/4$

41. (a) n^4 (b) $5^{100} - 1$ (c) $\frac{97}{300}$ (d) $a_n - a_0$

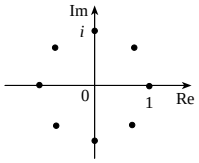
43. $\frac{1}{3}$ **45.** 14 **49.** $2^{n+1} + n^2 + n - 2$

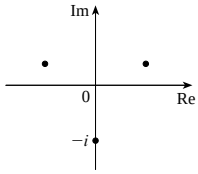
EXERCISES G ■ PAGE A54

1. (b) 0.405

EXERCISES H ■ PAGE A62

1. $8 - 4i$ **3.** $13 + 18i$ **5.** $12 - 7i$ **7.** $\frac{11}{13} + \frac{10}{13}i$
9. $\frac{1}{2} - \frac{1}{2}i$ **11.** $-i$ **13.** $5i$ **15.** $12 + 5i$, 13
17. $4i$, 4 **19.** $\pm\frac{3}{2}i$ **21.** $-1 \pm 2i$
23. $-\frac{1}{2} \pm \left(\sqrt{7}/2\right)i$ **25.** $3\sqrt{2}\,[\cos(3\pi/4) + i\sin(3\pi/4)]$
27. $5\left\{\cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right] + i\sin\left[\tan^{-1}\left(\frac{4}{3}\right)\right]\right\}$
29. $4[\cos(\pi/2) + i\sin(\pi/2)]$, $\cos(-\pi/6) + i\sin(-\pi/6)$, $\frac{1}{2}[\cos(-\pi/6) + i\sin(-\pi/6)]$
31. $4\sqrt{2}\,[\cos(7\pi/12) + i\sin(7\pi/12)]$, $\left(2\sqrt{2}\right)[\cos(13\pi/12) + i\sin(13\pi/12)]$, $\frac{1}{4}[\cos(\pi/6) + i\sin(\pi/6)]$
33. -1024 **35.** $-512\sqrt{3} + 512i$
37. ± 1, $\pm i$, $\left(1/\sqrt{2}\right)(\pm 1 \pm i)$ **39.** $\pm\left(\sqrt{3}/2\right) + \frac{1}{2}i$, $-i$





41. i **43.** $\frac{1}{2} + \left(\sqrt{3}/2\right)i$ **45.** $-e^2$
47. $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$, $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$